Section 4

Comparing Groups

Investigation 4.1
How Far Can You Jump?

Overview

This investigation focuses on students conducting a **comparative experiment** to explore the effect a fixed target will have on the distance students can jump from a starting line. Students will be randomly assigned to one of two groups. The first group will be asked to jump as far as they can from the starting line with no target in front of them. The second group will be asked to jump as far as they can, but a target (strip of tape) will be placed on the floor in front of them. Students will collect data about the distance jumped by each member of the two groups. They will display the data in a **back-to-back stemplot** or **boxplot**. Analysis of the data will include graphs and calculations of measures of center and spread.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Conduct an experiment to investigate a question
- Collect data and organize the results in a back-to-back stemplot (Level A) or side-by-side boxplots (Level B)
- Use the data to answer the question posed

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.
6. Attend to precision.

Common Core State Standards Grade Level Content

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.4 Display numerical data in plots on a number line, including dotplots, histograms, and boxplots.

6.SP.5 Summarize numerical data sets in relation to their context, such as by the following:

a. Reporting the number of observations

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6–8 Students should find, use, and interpret measures of center and spread—including mean and interquartile range—and discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stemplots, boxplots, and scatterplots.

Materials

- Masking tape
- Meter sticks
- Recording sheets (included on CD)
- Calculators

Estimated Time

1–2 days

Instructional Plan

Note: You may want to involve the physical education teacher in your school for assistance in this activity. This teacher can give suggestions regarding where to set the target line and how to collect the data.

✏ Formulate a Statistical Question

1. Ask your students if they know what a standing long jump is. Has anyone in class done a standing long jump before? Ask one student to demonstrate a standing long jump for the class. (Several short videos demonstrating the standing long jump are available on YouTube.) Share with your students that Norwegian Arne Tvervaag holds the world record for the standing long jump. He jumped 3.71 meters (12' 2.1") on November 11, 1968.

2. Discuss with your students some reasons why one student might jump farther than another. The following are some possible reasons students may come up with: height of a student, boys might jump farther than girls, what shoes they are wearing, whether there is a prize for the longest jump.

3. After students have generated their own ideas, ask them if they think setting a target line might help a student jump farther. This investigation discusses the statistical question, "Will students jump farther if they are given a fixed target in front of them?"

✏ Collect Appropriate Data

1. Before collecting data, there are procedures that need to be discussed with your students. It is important that your students are placed randomly into a group, that each student performs the jump in the same manner, and that the length of each jump is measured in the same way.

2. The generally accepted way to perform the standing long jump is to 1) stand with both feet up to the start line, 2) take a jump forward with both feet as far as you can, and 3) stay on your feet. **Note:** To avoid injury, this is best done on a mat or grass, instead of a hard floor.

3. The length of the jump should be measured from the start line to the part of the body that lands closest to the start line.

4. Ask students how the two groups should be formed. Students might suggest that there should be an equal number of boys and girls in each group, and some students will want to make sure the best athletes in class are spread between both groups. However, these designs do not ensure randomness. It is important that the groups are formed in a random manner. Random selection helps ensures that the two groups are similar in any attributes that might make a difference in performing the standing long jump. Discuss with your students how you might assign them randomly. One way to select students randomly is to write each of their names on an index card and then, after thoroughly mixing, draw one card at a time from the bag. The student named on the first card is assigned to the No Target group; the student named on the second card drawn is assigned to the Target group. Assignment of students continues to alternate until all the names have been drawn.

5. Set up two stations (one with No Target and one with a Target line) on the playground or in the gym where your students will perform the standing long jump. For the Target group, you may wish to ask the physical education teacher approximately how far your students will be able to jump. You want to set the target line toward the upper limit of what most students can jump. A suggestion for 12 year olds is 200 cm from the start line.

6. Each student in the No Target group will be asked to jump as far as she/he can from the starting position marked with tape on the floor. Following the jump, with a piece of masking tape, mark the location of the student's heel, or their hand if they fall backward. The heel or hand that is closest to the starting position should be used. Measure the distance in centimeters from the starting point to the end of the jump using a meter stick or extendable tape measure. Record the measurements on the data collection sheet. Similarly, each child in the Target group will be asked to jump as far as she/he can from the starting position marked with tape on the floor. Follow the same procedures as with the No Target group for marking, measuring, and recording the jump.

7. Collect the class data. Display each of the individual student results on the board under the headings No Target group and Target group. An example is shown in Table 4.1.1.

Table 4.1.1 An Example of Data Collected from a Group of 12 Year Olds

Length in Centimeters for No Target Group														
146	190	109	181	155	167	154	171	157	156	128	157	167	162	137
Length in Centimeters for Target Group														
199	167	147	180	185	170	171	139	154	126	179	158	181	152	

Note that the statistical design being followed is an independent groups one, in which each student participates in exactly one of the two treatments. Is this the best procedure to follow in the context of this problem? Be sure to read the extension and discuss it with your students after the experiment has been completed.

Analyze the Data

1. With the class data displayed on the board, ask your students if they think one group was able to jump farther than the other. Explain to your students that it is difficult to compare groups by just looking at the numbers; it is helpful to organize the data in a graph.

2. Have your students construct a back-to-back stemplot of the results. See Figure 4.1.1. On the board, label the No Target group on the left and the Target group on the right. The stems of the plot are the numbers 10–19, which represent 100 to 190. The "leaf" in the display represents the ones digit.

Jumping Length

No Target		Target
9	10	
	11	
8	12	6
7	13	9
6	14	7
7 6 7 4 5	15	4 8 2
2 7 7	16	7
1	17	0 1 9
1	18	0 5 1
0	19	9

Key: 16|7 represents 167 cm

Figure 4.1.1 Back-to-back stemplot comparing length of jumps for No Target group and Target group

3. Ask your students to modify their back-to-back stemplots showing the data (units digits) ordered. Figure 4.1.2 shows the back-to-back stemplot with the digits in order.

Jumping Length

No Target		Target
9	10	
	11	
8	12	6
7	13	9
6	14	7
7 7 6 5 4	15	2 4 8
7 7 2	16	7
1	17	0 1 9
1	18	0 1 5
0	19	9

Key: 16|7 represents 167 cm

Figure 4.1.2 Back-to-back stemplot comparing length of jumps for No Target group and Target group with the digits in order

4. Ask your students to compare the shapes of the two distributions from the stemplots. Note that the jump lengths in the No Target group are

concentrated between 150–170 cm, whereas those in the Target group are spread out a bit more and appear to be higher in length. The shape of the No Target distribution is peaked, while the shape of the Target distribution is more flat, uniform. There is a gap in the No Target group, suggesting that 109 cm might be what is called an *outlier*, an atypical value. The presence of an outlier might influence the most appropriate measure of center for the data set.

5. Ordering the digits in a stemplot is helpful when finding the quartiles (note the median is the second quartile). The three quartiles are used to construct another graph—the boxplot. To construct a boxplot, have your students find the five-number summary—minimum value, first quartile (Q1) that is the median of the data points strictly below the median of the distribution, the median, the third quartile (Q3) that is the median of the data points strictly above the median of the distribution, and the maximum value. Table 4.1.2 shows the five-number summary for both the Target group and No Target group. Figure 4.1.3 shows the side-by-side boxplots for the data in this example.

Table 4.1.2 Five-Number Summary for Target and No Target Group

	Min	Max	Median	Q1	Q3
No Target Group	109	190	157	146	167
Target Group	126	199	168.5	152	180

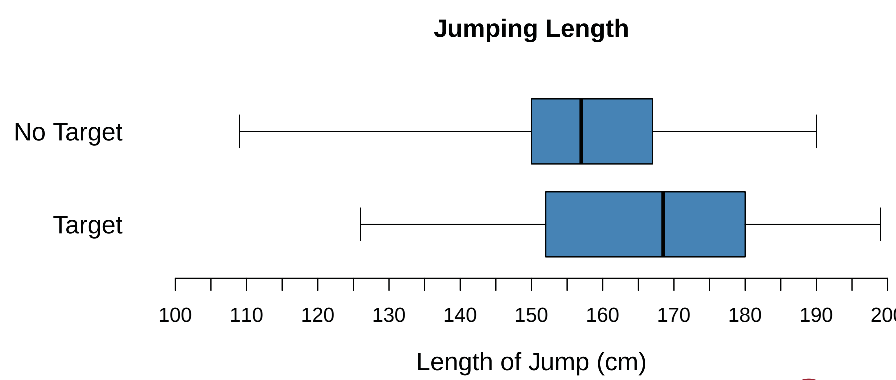


Figure 4.1.3 Side-by-side boxplot comparing length of jumps for No Target group and Target group

6. Remind your students that they are investigating whether a target helps or hinders the length of jumps. Ask your students to discuss several comparisons based on the two boxplots that will contribute to their final answer for the statistical question, "Will students jump farther if they are given a fixed target in front of them?" It is important to have your

students first discuss the meaning of the two boxplots. They should focus on comparing the medians, quartiles, and four sections of the boxplots. Note that in addition to the comparison of shapes they have made, they should note that the median for the Target group is 11.5 cm higher than that for the No Target group. That's a considerable distance. A related note to that comparison of medians is that although the first quartiles are somewhat similar (meaning that 75% of the students in each group jumped at least somewhere around 150 cm), half the students in the Target group jumped more than 168.5 cm, but half the students in the No Target group jumped no more than 157 cm, 11.5 cm shorter. Even more telling is that half the Target group jumped farther than 75% of the No Target group (target median is 168.5, no target Q3 is 167).

7. In addition to graphing and finding the median and quartiles, ask your students to find another measure of center—the mean length of the jumps. Table 4.1.3 (template available on the CD) shows the sample data and five-number summary and the mean. Discuss with your students whether to use the mean or median. The median is more robust in that it is less influenced by extreme values. The mean is influenced by extreme values, but includes all the information in the calculation. In this example, it appears that 109 is an extreme value in the No Target Group, so the median might be a better measure of center than the mean for the No Target group. Note that whichever measure is used, it should be the same for comparison purposes.

Table 4.1.3 Example Recording Sheet

Student Number	Group 1 - No Targeted Jump (cm)	Group 2 - Targeted Jump (cm)
1	146	199
2	190	167
3	109	147
4	181	180
5	155	185
6	167	170
7	154	171
8	171	139
9	157	154
10	156	126
11	128	179
12	157	158
13	167	181
14	162	152
15	137	
Summary Measures		
Mean	155.8	164.8
Median	157	168.5
Minimum	109	126
Maximum	190	199
Q1	146	152
Q3	167	180

8. Statistics is the study of variability, so a measure of spread needs to be computed to better compare the two groups. Discuss with your students that they calculated one measure of variability when they drew their boxplots, the interquartile range (IQR). The IQR = Q3 – Q1, the difference between the 1st quartile and the 3rd quartile. The IQR provides a measure of the spread of the middle 50% of the jump lengths. In the example data, the IQR of the No Target group is 21 and the IQR for the Target group is 28. This means the middle 50% of the jump lengths for the Target group has a greater spread than the middle 50% of the jump lengths for the No Target group. Discuss with your students what conclusion can be drawn about a data set concerning how spread out it is. Note that a compact data set makes its center more believable that it is reflecting the true value, whereas a widely dispersed data set makes us less sure the center is really characterizing typical performance.

   Have your students compare the two IQRs in words in the context of the data (i.e., what do the IQRs say about how spread out the jump lengths are in the No Target group compared to the Target group). Have them provide a possible contextual explanation as to why they are different. Suggestions will vary. One possibility is that in the presence of a target, people react differently. Some tense up and others push themselves beyond their normal performance.

9. Recall that from the stemplot for the No Target group, 109 was thought to be a possible outlier because it was separated from the rest of the data by a gap. The boxplot allows for a more formal determination as to whether a value should be labeled an outlier (extreme value). The procedure is to calculate what are called the upper fence and lower fence. Data points outside the fences are considered outliers (i.e., data atypical to the data set). The upper fence is Q3 + 1.5*IQR; the lower fence is Q1 – 1.5*IQR. Ask your students to calculate the fences for the No Target group. Note that the lower fence is Q1 – 1.5*IQR = 146 – 1.5*(167 – 146) = 114.5. So, it can be concluded that 109 is an outlier. The implication of this is that, in a statistical analysis of this No Target data set, it would be advisable to use the median as a measure of its center, rather than the mean.

✏ Interpret the Results in the Context of the Original Question

1. Have your students recall the original question, "Will students jump farther if they are given a fixed target in front of them?" Ask your students to write a summary of the experiment that starts with stating an answer

to the question and then supporting their answer with their analysis. They should focus their summary on using center and spread measures, but also include a discussion about the shapes of the graphs they drew.

2. Have your students describe what they think the distribution of jumps with and without a target would be if 2nd graders performed the experiment. Do they think their conclusion they reached about the effect of a target line will be the same for the 2nd graders?

Example of 'Interpret the Results'

Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.

We conducted a comparative experiment in which some students did a standing long jump with no target in front of them and others did a standing long jump with a target 200 cm in front of them to answer the statistical question, "Will students jump farther if they are given a fixed target in front of them?" (Our gym teacher suggested 200 cm would be a good target for 12 year olds.)

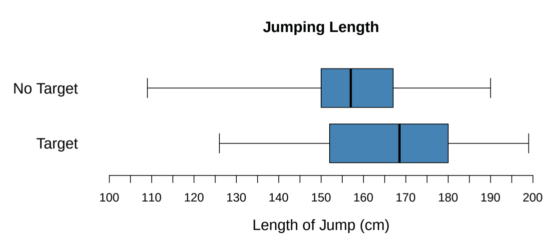
To determine which of us would be in the No Target group and which would be the Target group, we put our names in a hat. The first name randomly drawn from the hat was assigned to the No Target group. The second name drawn was assigned to the Target group. We went back and forth like that until everyone had been assigned to a group.

We measured our distances in centimeters from the starting line to where the closer heel of our shoes landed to the start line. (Everyone landed on their feet.) We tried to make sure everyone did the jump the same way to avoid introducing any sort of bias, like measurement bias, into our results. We drew two comparative graphs of our data.

Jumping Length

No Target		Target
9	10	
	11	
8	12	6
7	13	9
6	14	7
7 7 6 5 4	15	2 4 8
7 7 2	16	7
1	17	0 1 9
1	18	0 1 5
0	19	9

Key: 16|7 represents 167 cm



From the stemplot—except for one possible outlier (109) in the No Target group, it looked like the data sets were spread about the same. But the IQR for the No Target group is 21 and a larger 28 for the Target group, so the middle 50% of the no target data is more compact than for the Target group.

Actually, it's better for a data set to have a small variation because it makes us more confident about the centering value. We thought the target group should be more compact because those jumpers had something to concentrate on, but it didn't turn out that way. Regarding the 109, it is an outlier looking at the gap in the stemplot, and it is also an outlier using the Q1 - 1.5*IQR rule for the boxplot. Any value below 146 – 1.5 * (167 – 146) = 114.5 is considered an outlier.

So, did those in the Target group jump farther than the No Target group? From the stemplots, the Target group is shifted to the right compared to the No Target group. Because the No Target group has an outlier, we decided to compare the two groups with medians, rather than means. Based on medians, the answer would be yes, since the median for the Target group was 168.5 cm compared to the median for the No Target group of 157 cm. The Target group jumped a full 11.5 cm longer. In fact, half (seven students) of the Target group jumped farther than 168 cm, but only 3 of the 15 No Target group (20%) jumped that far. Having a target produces higher standing long jump distances. We were wondering if the same conclusion would be made for other age groups. Our guess is that no matter what age groups do this experiment, the results will be similar, since it seems better to have a target as a goal to achieve.

Assessment with Answers

A group of students conducted an experiment to compare the effect of where the target line is placed for the standing long jump. Target lines were placed at 100 cm and 300 cm. Table 4.1.4 shows the length of the jumps in cm for each group.

Table 4.1.4 Jump Lengths (cm) for Groups with Target of 100 cm and 300 cm

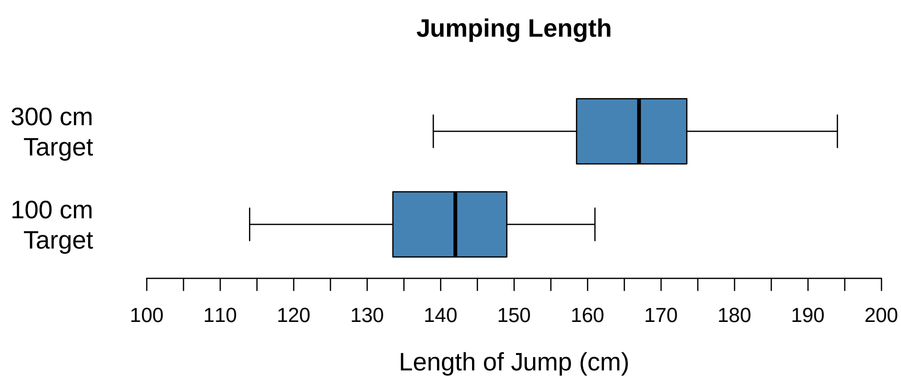
100 cm Target	149	141	161	114	116	142	129	149	138	158	145
300 cm Target	168	185	194	167	147	151	169	178	167	166	139

1. Does the distance a target line is from the start line affect the distance students jump in the standing long jump? Yes, students tended to jump farther when the target line was set at 300 cm.

2. Use words, numbers, and graphs to justify your answer by using at least one graph, a measure of center, and a measure of spread.

Summary

	100 cm target	300 cm target
Mean	140.2	166.5
Minimum	114	139
Q1	129	151
Median	142	167
Q3	149	178
Maximum	161	194
IQR	20	27



Students tended to jump farther when the target line was set at 300 cm than at 100 cm. The mean jumping distance for the 300 cm target was 166.5 cm, while the mean for the 100 cm target was 140.2. The boxplot of the 300 cm target group is shifted much further right than the 100 cm target group. About 75% of the data in the 300 cm target group are greater than about 75% of the 100 cm group.

Extensions

1. As mentioned earlier, the procedure used with all students knowing the experimental condition will no doubt bias the results, as those not assigned to the Target group may imagine a target line. To avoid this potential introduction of bias into the model, redesign the experiment using a **matched pairs** design. Each student does the standing long jump at both stations and the difference—target jump distance minus the no target jump distance—is noted between the two jumps. Your students

should be assigned randomly to which jump they do first. Your students will analyze the differences by making a dotplot, stemplot, or boxplot. If the differences are generally greater than zero, then target jump distances were better than no target distances.

2. Another measure of spread is the mean absolute deviation (MAD), found in Common Core Standard 6.SP.5c (see Investigation 3.4). Calculate the mean absolute deviation (MAD) for each group and compare the two MADs in words in the context of the experiment.

The MAD is the average of the absolute values of the distances from the group's mean. "Deviation" refers to the difference a value is from the mean. "Absolute deviation" is the absolute value of that difference. Column one of Table 4.1.5 contains the data; column two lists the data minus the mean (the deviation); and column three has the absolute value of the deviations in column two. To find the MAD, find the mean of the values in column three.

Table 4.1.5

No Target	No Target – Mean	|No Target – Mean|
146	146 - 155.8 = -9.8	9.8
190	190 - 155.8 = 34.2	34.2
109	109 - 155.8 = -46.8	46.8
181	181 - 155.8 = 25.2	25.2
155	155 - 155.8 = -0.8	0.8
167	167 - 155.8 = 11.2	11.2
154	154 - 155.8 = -1.8	1.8
171	171 - 155.8 = 15.2	15.2
157	157 - 155.8 = 1.2	1.2
156	156 - 155.8 = 0.2	0.2
128	128 - 155.8 = -27.8	27.8
157	157 - 155.8 = 1.2	1.2
167	167 - 155.8 = 11.2	11.2
162	162 - 155.8 = 6.2	6.2
137	137 - 155.8 = -18.8	18.8

The sum of the absolute deviations in this example for the no target data is the sum of the third column, namely 211.6. Dividing the sum by the number of values, 15, yields the mean of 14.1. In words, the average distance away from 155.8 cm that the 15 students jumped was 14.1 cm for the no target group.

Similarly, the MAD for the target group is 16.2 cm. So, according to the point of view of average distance data are from its mean, the target data are spread out more from their mean than the no target data are from their mean. Ask your students if that result is reflected in their boxplots. Why?

References

Franklin, C., G. Kader, D. Mewborn, J. Moreno, R. Peck, M. Perry, and R. Scheaffer. 2007. *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-k–12 curriculum framework*. Alexandria, VA: American Statistical Association. www.amstat.org/education/gaise.

National Council of Teachers of Mathematics. 2000. *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Common Core State Standards for Mathematics. www.corestandards.org.

INVESTIGATION 4.2
HOW FAST CAN YOU SORT CARDS?

Overview

Students are always interested in how fast they can do something such as playing video games, texting, or running a race. This investigation focuses on the use of a comparative experiment to investigate possible differences in the average time it takes a student to sort a set of 10 cards in numerical order when the size (number of digits) in the numbers varies. Students will be **randomly assigned** to one of three groups. Students in Group 1 will each sort a deck of cards labeled with two-digit numbers. Students in Group 2 will each sort a deck of cards labeled with three-digit numbers. Students in Group 3 will each sort a deck of cards labeled with four-digit numbers. A stopwatch will be used to measure the time needed to complete the task. Students will compare the summary from each group using **measures of center** (mean and median) and **variability** (range, interquartile range, mean absolute deviation) and graphically compare the results using stemplots and boxplots. An informal inference procedure will be introduced as suggested by the Common Core State Standards. This investigation is focused on providing an answer to "Does the time it takes to sort a deck of digit cards vary with the number of digits in the numbers?"

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
- Explain the idea and use of random assignment
- Conduct an experiment to investigate questions

- Use the data to answer the questions posed
- Collect data and organize the results into stemplots and boxplots
- Compare the results from each group using summary measures of center (such as mean and median) and measures of variability (such as range and interquartile range)

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Common Core State Standards Grade Level Content

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

6.SP.2 Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.

6.SP.3 Recognize that a measure of center for a numerical data set summarizes all its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.4 Display numerical data in plots on a number line, including dotplots, histograms, and boxplots.

6.SP.5 Summarize numerical data sets in relation to their context, such as by the following:

a. Reporting the number of observations
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from

the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

7.SP.3 Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities measuring the difference between the centers by expressing it as a multiple of a measure of variability.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6–8 In grades 6–8, all students should find, use, and interpret measures of center and spread—including mean and interquartile range—and discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stemplots, boxplots, and scatterplots.

Materials

- Three sets of numbered cards (template available on the CD)
- Recording sheets (available on the CD)
- Stopwatches or other timing devices (need to be able to time to nearest 1/10 of a second)

Estimated Time

1–2 days

Instructional Plan

✏ Formulate a Statistical Question

1. Begin the investigation by asking your students when they sort items and what items they sort. Ask if they ever sort numbers in their mathematics class. When finding the median of a set of data, the data must be arranged in order. Tell your students this investigation focuses on sorting cards with numbers on them. Explain that one deck consists of two-digit

numbers, a second with three-digit numbers, and a third with four-digit numbers. Show your students one of these decks of cards. Ask them for factors that may influence how fast they can sort the cards from lowest to highest. They may suggest factors such as the size of the numbers (i.e., the number of digits in the number), the underlying sequence of the numbers, the number of cards, and any incentive offered such as whether there is a prize for the fastest time.

2. Help your students write their suggested factors in the form of a statistical question. This investigation addresses the statistical question, "Does the time it takes to sort a deck of digit cards vary with the number of digits in the numbers?"

✏ Collect Appropriate Data

1. Introduce the idea of comparing the results from three groups of students, each group doing a different version of the task. This is an example of an experiment.

2. Ask students what the variables are in this investigation. Students should realize the first variable of interest is the experimental group (two, three, or four digits) and the second variable of interest is the amount of time needed to complete the task of sorting the cards (as measured in seconds).

3. Discuss with your students the methods they use to select teams on the playground. Are the methods fair? Does each student have the same chance (opportunity) to be selected? What method should we use to assign students to each of the three groups? One way to select students randomly is to write each of their names on an index card and then, after thoroughly mixing, draw one card out of a bag at a time. The student named on the first card is assigned to the Deck 1 Group; the student named on the second card drawn is assigned to the Deck 2 Group; the student named on the third card drawn is assigned to the Deck 3 Group. Assignment of students continues in this pattern until all the names have been drawn.

   Note that some students may suggest that each student roll a die. If a 1 or 2 comes up, the student uses deck 1; 3 or 4, deck 2; 5 or 6, deck 3. Ask them why this method is not desirable. (We should have about the same number of students assigned to the three decks, but it is possible that, in the extreme, all students roll a 5 or 6, say, using this die method.)

4. Point out that in order for them to truly be able to make comparisons, they need to make sure time is measured in the same way for all participants. Therefore, they all need to use the same type of stopwatch and give careful attention to the beginning and ending of the task. Note that one person should do the timing for each student in the specific group (i.e., two-digit, three-digit, or four-digit) to avoid some of the measurement variability.

5. Within each of the three groups, select a member of the group to serve as the timer (leader). Students will perform the sorting task one at a time within each group. Before each student in the group begins, the leader will shuffle the deck of cards (template of cards available on the CD), hand them to a student, and say "GO" and start the stopwatch. The student will sort the cards in ascending order from lowest to highest and say "DONE" when completed. At that time, the leader will stop the stopwatch and record the time on the data collection form.

6. If a student sorts the numbers in the wrong order, the timer should not stop the watch until the numbers are in the correct order from lowest to highest.

7. Collect the class data on the data collection sheet. See Table 4.2.1 for an example.

Table 4.2.1 Example of Class Data

Time (sec) to Sort 2 Digits	Time (sec) to Sort 3 Digits	Time (sec) to Sort 4 Digits
20.6	26.2	31.2
22.9	25.8	28.6
20.9	24.1	28.3
22.2	24.3	31.3
25.6	25.9	26.8
23.1	24.4	27.9
19.6	26.4	28.9
23.6	29.5	27.2
20.5	28.4	34.3
22.0	25.1	26.2
21.8	24.0	25.2

Analyze the Data

1. Begin the analysis by having your students make observations about the class data such as almost all of the data are between 20–30 seconds.

2. Suggest to your students that more observations can be made from graphs. Have your students make a stemplot of each of the three sets of times. See Figures 4.2.1, 4.2.2, and 4.2.3 for examples.

Sort Times for 2-Digit Numbers

19	6
20	5 6 9
21	8
22	0 2 9
23	1 6
24	
25	6

Key: 25|6 represents 25.6 sec.

Figure 4.2.3 Stemplot of sort times for 2-digit numbers

Sort Times for 3-Digit Numbers

24	0 1 3 4
25	1 8 9
26	2 4
27	
28	4
29	5

Key: 26|2 represents 26.2 sec.

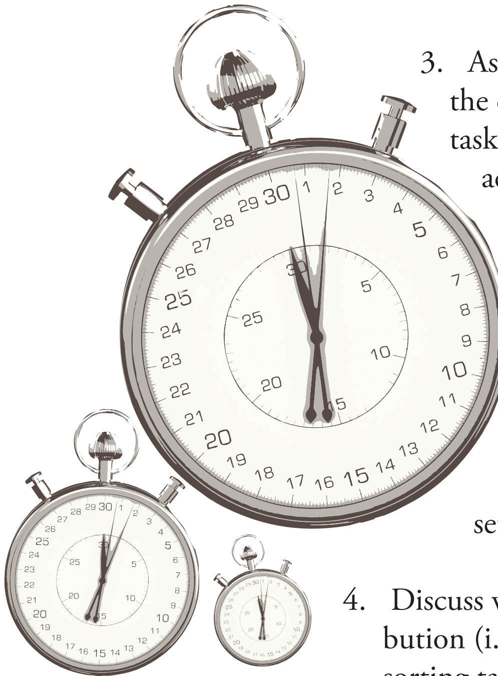
Figure 4.2.3 Stemplot of sort times for 3-digit numbers

Sort Times for 4-Digit Numbers

25	2
26	2 8
27	2 9
28	3 6 9
29	
30	
31	2 3
32	
33	
34	3

Key: 31|2 represents 31.2 sec.

Figure 4.2.3 Stemplot of sort times for 4-digit numbers



3. Ask your students for some observations from the plots regarding the effect the number of digits has on the time to do the ordering task. Note that they should compare shapes: 2-digit might be characterized as bi-modal, 3-digit as "ski-sloped" skewed to the right, and 4-digit as kind of mound-shaped but with big gaps. Each graph shows gaps, but especially 4-digit. Students have to be careful in that the scales of the stemplots are not the same. Drawing dotplots on the same scale would definitely show that 2-digit is to the left of the other two, with 4-digit drifting to the right. The gaps indicate the presence of potential extreme values called outliers. Outliers need to be identified because they can influence conclusions made about the data set, particularly regarding the center.

4. Discuss with your students methods to summarize the center of a distribution (i.e., what could be a representative time needed to complete the sorting task in each group?). Students should suggest that they could use either the mean or the median. Have them do the calculations and then discuss if one measure is more representative of the center of the data in each group than the other and why they think that way. Note that, for these data sets, the respective medians are 22.0, 25.8, and 28.3; the respective (rounded) means are 22.1, 25.8, and 28.7. The medians and means are very close to each other in each group, so either could be used to measure center. Note that the presence of potential outliers in the data sets did not influence the mean as is often the case. Ask your students to look at each data set to see why the medians and means were comparable.

5. Ask your students to comment on what the means or medians are telling them about the typical time taken to complete the task in each group. Note that it's clear the two-digit group is, on average, the quickest, followed by the three-digit group and the four-digit group coming in the slowest.

6. Ask your students whether the overall distributions are the same since their means and medians are about the same in each case. Discuss with them that distributions are compared by their centers and variability. Discuss ways to measure the variability in the data. The range is a basic measure of spread. Recall that the range is the maximum value minus the minimum value. For these data, respectively, the ranges are 6.0, 5.5, and 9.1. Have your students discuss that the first two groups are somewhat similar in how spread out their data are, whereas the third group contains considerably more spread. Looking at the actual data in the stemplots,

discuss gaps and the reason the spread in the third group is so wide. Note that it is due to 34.3 being so much higher than the rest of the group.

7. Another measure of variation is the interquartile range (IQR) that is the third quartile (Q3) minus the first quartile (Q1). Recall that Q1 is the median of the data points strictly below the median of the distribution. Q3 is the median of the data points strictly above the median of the distribution. Note that the IQR focuses on the middle 50% of a distribution, whereas the range measures the entire distribution from lowest to highest. Have your students calculate Q1, Q3, and the IQR for each group. Referring to the IQRs, discuss how the variations in the groups compare. Also, discuss how conclusions about variation might differ depending on whether the IQR or the range is used. See Table 4.2.2 for a summary of the calculations.

Table 4.2.2 Five-Number Summary for Each Group

	Min	Max	Range	Q1	Q3	IQR
Two-Digit Group	19.6	25.6	6.0	20.6	23.1	2.5
Three-Digit Group	24.0	29.5	5.5	24.3	26.4	2.1
Four-Digit Group	25.2	34.3	9.1	26.8	31.2	4.4

8. Have students construct side-by-side boxplots. See Figure 4.2.4.

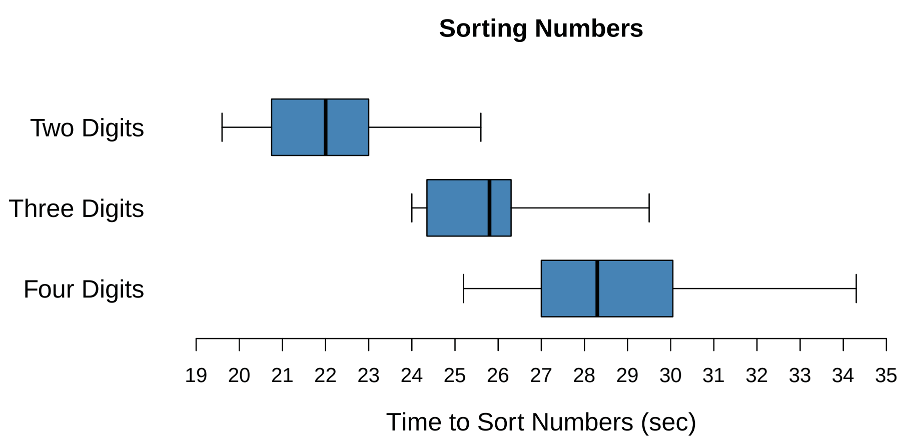


Figure 4.2.4 Side-by-side boxplots of the example class data

9. Ask your students what observations they can make from the boxplots. In particular, is their median measure of center reflected in the boxplots as well as their measures of spread, range, and inter-quartile range? Discuss how. Note that the boxplots make it clear that the medians are increasing, that the IQR in 2-digit and 3-digit are similar, and that IQR

for 4-digit is about twice as much. All the 3-digit and 4-digit values were higher than 75% of the 2-digit. Seventy-five percent of the 4-digit were higher than 75% of the 3-digit. The 3-digit median exceeded all the 2-digit times.

10. Have your students look at the two types of graphs they have constructed—stemplots and boxplots—and discuss what each of the plots reveal and don't reveal about the comparison of the groups. Lead them to the discovery that several types of graphs should be displayed in a statistical investigation, since each looks at a set of data from a different point of view. Putting all the information together enables the viewer to get a more complete understanding of the experimental results. For example, ask your students if the potential outliers as indicated by the gaps in the stemplots are evident in the boxplots. In boxplots, to identify potential outliers, two calculations need to be made. They are called the lower fence and the upper fence. Values in the data set outside the fences are identified as outliers. The lower fence is Q1 – 1.5*IQR, and the upper fence is Q3 + 1.5*IQR. Have your students calculate the fences for each data set and determine if there are any outliers according to this rule. There are none.

Group	Q1	Q3	IQR	1.5*IQR	Lower Fence	Upper Fence	Outliers
2-digit	20.6	23.1	2.5	3.75	16.85	26.85	None
3-digit	24.3	26.4	2.1	3.15	21.15	29.55	None
4-digit	26.8	31.2	4.4	6.60	20.20	37.80	None

11. In statistical inference, to determine if the centering points of two distributions are statistically close or far apart, their difference is written in terms of the number of units of some measure of variation. Then, that number of units is determined by various techniques to conclude whether the difference of means is small or large (statistically significant). There is a technique your students will be doing as part of the Common Core State Standard in statistics and probability for all high-school students. (There is a formal technique that students who take Advanced Placement Statistics will learn.)

    The Common Core State Standard 7.SP.3 introduces middle-school students to an informal inference procedure by having them measure how far apart two medians or two means are in terms of the number of units of a measure of variability such as IQR. The two distributions being compared have to be of similar variability, and it is the common value that is used to measure how far apart the centers are. Have your students compare the 2-digit and 3-digit distributions. Recall that the mean of the 2-digit data set is 22.1 cm and the mean of the 3-digit data set is 25.8 cm. The two IQRs are 2.5 and 2.1, which are fairly close. Let's be conservative and take the maximum 2.5 to represent the common spread of the two distributions. By how many IQRs of 2.5 cm do the means 22.1 and 25.8 differ? The means of the 2-digit and 3-digit data sets differ by (25.8 – 22.1) / 2.5= 1.5 IQRs.

✏ Interpret the Results in the Context of the Original Question

Have your students recall the original question, "Does the time it takes for a deck of digit cards to be sorted vary with the number of digits in the numbers?" Have your students write a summary of the experiment based on the data collected and analyzed that answers the original question (i.e., what group do they think sorted the cards the fastest and why). They need to support their answer by including the following:

a. A discussion of the plan they used to collect the data

b. The graphs they drew and conclusions made from looking at them

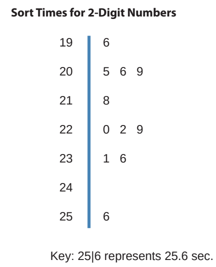
c. The measures of center and variability they computed

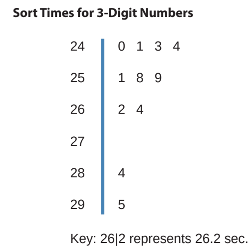
d. What the measures said about the comparison of the groups (e.g., whether the measures were similar from group to group).

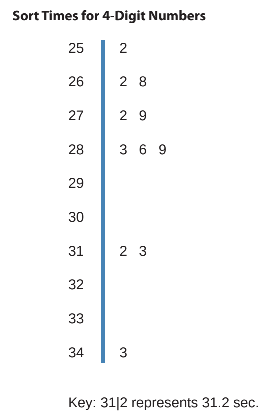
Example of 'Interpret the Results'

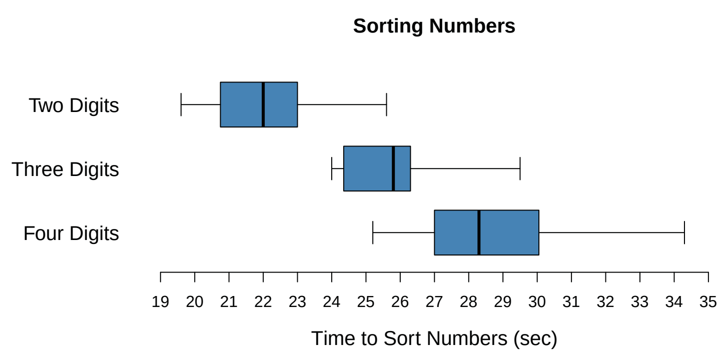
Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.

We investigated how fast it took us to sort cards that had two-, three-, or four-digit numbers on them. There were 17 cards in each group. We were assigned to one of the groups. To avoid introducing bias into the experimental procedure, we put all our names in a container and then drew them out randomly, one at a time, assigning the first name to the two-digit group, the second to the three-digit group, and the third to the four-digit group. We repeated this until everyone was assigned. After getting our data, we drew stemplots and boxplots.









Each stemplot had at least one gap, indicating there were possible outliers. The two-digit shape had a dip in the middle, but looked symmetric. The three-digit shape was definitely skewed to the right. The four-digit one looked like a triangle for the lower values and then had a couple big gaps. We should have put the stemplots side by side on the same scale like we did with the boxplots. It was really clear from the boxplots that the medians increased and the spread of the middle 50% measured by IQR of the 2-digit and 3-digit data sets was similar, with the spread of the 4-digit about twice as much. We saw many comparisons such as all the 3-digit and 4-digit times were longer than 75% of the 2-digit times. The median of 3-digit exceeded all 2-digit. So, overall, it was clear that the times to sort the cards are longer as the number of digits in the numbers increases.

It was interesting that the medians (22.0, 25.8, and 28.3) were about the same as the means (22.1, 25.8, 28.7) even though the distributions had all those gaps. We guessed the possible outliers kind of balanced out the distributions. We checked to see if the outliers we saw in the stemplots were also outliers by the 1.5*IQR calculation for boxplots and none were. Different graphs illustrate different things. Finally, we compared the means of the 2-digit and 3-digit groups by calculating how many common IQRs separated them. We used the maximum IQR of 2.5 for the value of the IQRs and saw that the means 22.1 and 25.8 differed by (25.8 – 22.1)/2.5 = 1.5 IQRs. We don't really have a number to compare 1.5 to, but it seems to us that 1.5 IQRs is large enough to say the means differ from each other, since they are really seperated when we look at the boxplots.

Assessment with Answers

A class of sixth- grade students conducted an experiment involving LEGO blocks to compare the effect of the type of directions provided to a student on the time needed to complete a task. The task was to build a tower from a given set of blocks. A bag of LEGO blocks contained one of the following three sets of directions:

> **Directions Set 1:** Construct a tower using all the blocks in this bag. The longest blocks should be on the bottom and go up in order to the shortest LEGO blocks at the top.
>
> **Directions Set 2:** Construct a tower using all the blocks in this bag according to the picture. (Figure 4.2.5)
>
> **Directions Set 3:** Build a tower with the blocks.

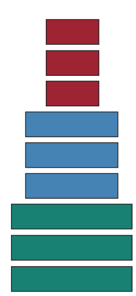


Figure 4.2.5 Diagram shown on directions for set 2

The class was randomly divided into three groups; the results of the experiment are shown in table 4.2.3.

Table 4.2.3 Time to Build Tower

Time (sec) to Build Tower with Directions for Set 1	Time (sec) to Build Tower with Directions for Set 2	Time (sec) to Build Tower with Directions for Set 3
18.1	22.1	11.6
17.5	21.3	15.5
16.3	18.9	15.4
18.8	19.5	15.6
16.2	20.1	15.3
16.0	21.0	15.7
16.6	19.4	13.8
14.8	16.5	16.1
18.1	22.7	15.9
19.8	19.1	16.8
17.6	21.6	14.3
16.5	20.0	12.9
16.7	20.0	17.0

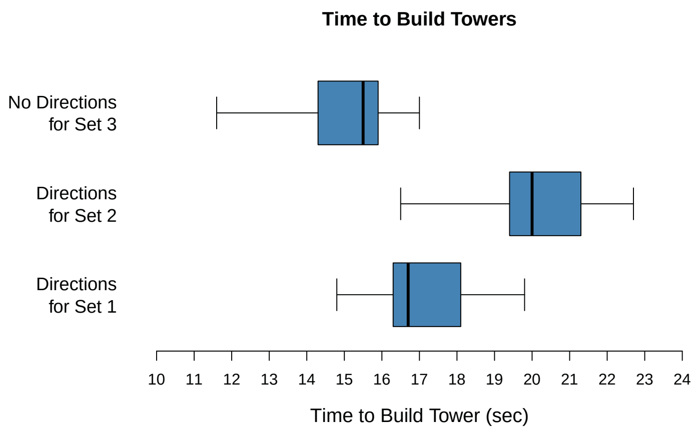
1. What is an appropriate statistical question in the context of this study?
   Does the average time it takes to build a tower with blocks vary with the type of directions given?

2. Find the mean for each group. Set 1 mean = 17.1 sec. Set 2 mean = 20.2 sec. Set 3 mean = 15.1 sec.

3. Find the five-number summary for each of the groups.

	Set 1	Set 2	Set 3
Minimum	14.8	16.5	11.6
Q1	16.3	19.3	14.1
Median	16.7	20.0	15.5
Q3	18.1	21.5	16.0
Maximum	19.8	22.7	17.0

4. Construct side-by-side boxplots of the three groups.



5. Which of the three groups was able to build the tower faster? Using words, numbers, and graphs, explain why you chose the group you did.
   Group 3 was able to build the tower the fastest. The median of this group is less than the other two. About 75% of the times for Group 3 are less than all of Group 2 times and 75% of Group 1.

Extension

1. Vary the background of the cards. Using a standard deck of playing cards, create three stacks. Each stack contains the cards ace to 10 with one stack having cards that are all of the same suit, one stack having cards from the two black suits, and one stack having mixed red and black suits. Students would investigate the statistical question, “Does the mixture of suits of cards relate to the amount of time needed to place the cards in order?”

2. Consider Step 11 of the Analysis of the original question in this investigation. Instead of calculating how many common IQRs separate two means, the separation also can be calculated in terms of MADs. Note that the MADs for each group have to close in value so a common value can be determined. Ask your students to calculate mean absolute deviations for the three groups to see if any are similar and, if so, to do Step 11 using MAD in place of IQR.

References

Franklin, C., G. Kader, D. Mewborn, J. Moreno, R. Peck, M. Perry, and R. Scheaffer. 2007. *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-k–12 curriculum framework*. Alexandria, VA: American Statistical Association. www.amstat.org/education/gaise.

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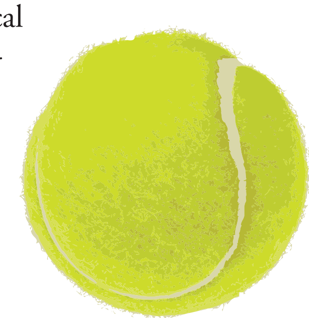
Investigation 4.3
How High Does a Ball Bounce?

Overview

This investigation focuses on students conducting an experiment to determine the bounce height two kinds of balls will reach when dropped from various heights. Students will collect data using a tennis ball and a golf ball that will be dropped from 30, 60, and 90 cm. They will display the data in a **scatterplot** and interpret the results to answer the statistical question, "Does the height from which a ball is dropped affect how high it bounces?"



Note: The Common Core State Standards do not specifically address measurement error, but this experiment has many areas in which error can occur and that can lead to increased variability within height groups. It could be a topic of extended discussion.

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Pose investigative questions
- Design and conduct an experiment to investigate questions
- Collect data by conducting an experiment and organize the results in a scatterplot
- Recognize linear relationships and use that information to interpret the data

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

Common Core State Standards Grade Level Content

6.SP.1 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

6.SP.5 Summarize numerical data sets in relation to their context, such as by doing the following:

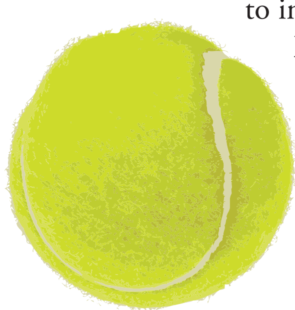
a. Reporting the number of observations

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

8.SP.1 Construct and interpret scatterplots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.



6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

NCTM Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6–8 All students should find, use, and interpret measures of center and spread—including mean and interquartile range—and discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stemplots, boxplots, and scatterplots.

Materials

- Two types of balls that bounce well (e.g., tennis ball, golf ball, basketball, or table tennis ball)
- Meter sticks
- Calculators
- Recording sheet for each type of ball (available on the CD)
- Graph paper

Estimated Time

Two days

Instructional Plan

✏ Formulate a Statistical Question

Note: This investigation will use a tennis ball and golf ball, but any two types of balls can be used.

Begin by holding a tennis ball and golf ball up for the students. Have them generate questions about the differences between a tennis ball and golf ball. The following are some of the questions students may come up with:

How are they made?

Which one weighs more?

What are they made out of?

Which ball could you throw farther?

Which ball bounces higher?

Have students generate their own statistical questions. This investigation focuses on two questions: "Does the height from which a ball is dropped affect how high it bounces?" and "Do tennis balls bounce higher, lower, or the same as golf balls?"

Collect Appropriate Data

1. Ask your students how they think the data should be collected.

2. Point out that to make comparisons, dropping balls must be done in the same way and onto the same type of surface (i.e., experimental conditions must be the same).

3. Following is the procedure for the experiment:

   a. Divide your students into groups of three. One person will drop the ball, a second will observe the height of the bounce, and a third will record the results in a table. Discuss why it would be beneficial to have the same student doing all the ball bouncing and the same student doing the measuring for all heights.

   b. Tape the meter stick to the wall with the 1 cm end on the floor and the 100 cm end at the top.

   c. Hold the tennis ball so that the bottom of the ball is at the 30 cm mark. Drop the ball; don't "throw" the ball down.

   d. Watch carefully to see how high it bounces back up. Record the height in Table 4.3.1. Repeat the drop two more times, recording each trial. Note that students may find it difficult to gauge the height accurately. The ball bounces back very fast. You may want to have two students watch the height of the bounce and compare their numbers. They need to agree or the drop is repeated.

   e. Next, drop the tennis ball from 60 cm three times. Record the height of each of the three bounces in Table 4.3.1. Repeat for a 90 cm height.

Table 4.3.1 Tennis Ball Recording Sheet

	Tennis Ball		
	30 cm	60 cm	90 cm
Trial 1			
Trial 2			
Trial 3			

4. Repeat the tennis ball procedure with a golf ball. Drop a golf ball three times from 30, 60, and 90 cm. Each time, record the height of the bounce. Record the data in a table similar to Table 4.3.2.

Table 4.3.2 Golf Ball Recording Sheet

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1			
Trial 2			
Trial 3			

5. Tables 4.3.3 and 4.3.4 contain data collected by a group of students.

Table 4.3.3 Example of Tennis Ball Bounce Height

	Tennis Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	16 cm	33 cm	50 cm
Trial 2	17 cm	32 cm	49 cm
Trial 3	16 cm	33 cm	49 cm

Table 4.3.4 Example of Golf Ball Bounce Height

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	19 cm	37 cm	55 cm
Trial 2	17 cm	32 cm	49 cm
Trial 3	16 cm	33 cm	49 cm

6. Ask your students what the variables are in this investigation. Students should realize that the first variable of interest is the type of ball (tennis versus golf ball) and the second variable of interest is the height of drop (30, 60, 90 cm).

7. Ask your students why they think they had to drop the balls from each height three times? Students should realize that by taking more measurements, the final heights could be more accurate.

Analyze the Data

1. Ask your students how the information gathered for each ball at each height can be consolidated. For example, we dropped the tennis ball from 30 cm three times. How can we determine a representative height for the bounce of the tennis ball from 30 cm? Students should suggest that they could use either the mean or the median.

2. Discuss with your students whether to use the mean or median. The median is more robust in that it is less influenced by extreme values. The mean is influenced by extreme values, but includes all the information in a calculation. Students should realize that both measures of center might be valuable here.

3. Have your students calculate the mean for their three drops for each ball at each height. Have them find the median for their three drops for each ball at each height. Students should record the mean and median on their recording sheet. Table 4.3.5 and Table 4.3.6 show results from an example experiment.

Table 4.3.5 Sample Results for Tennis Ball Drop

	Tennis Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	16.0 cm	33.0 cm	50.0 cm
Trial 2	17.0 cm	32.0 cm	49.0 cm
Trial 3	16.0 cm	33.0 cm	49.0 cm
Mean	16.3 cm	32.6 cm	49.3 cm
Median	16.0 cm	33.0 cm	49.0 cm

Table 4.3.6 Sample Results for Golf Ball Drop

	Golf Ball		
	30 cm	**60 cm**	**90 cm**
Trial 1	19.0 cm	37.0 cm	55.0 cm
Trial 2	18.0 cm	38.0 cm	57.0 cm
Trial 3	19.0 cm	38.0 cm	56.0 cm
Mean	18.6 cm	37.6 cm	56.0 cm
Median	19.0 cm	38.0 cm	56.0 cm

4. Have each group of students construct a scatterplot of their results. Note that it is customary to put the independent variable on the x-axis, which is the height of the drop for this experiment. The variable on the y-axis should be the height of the bounce (since this is the dependent variable—dependent upon the height of the drop). Instruct your students to graph the median bounce height for each drop height. They should graph the data for both types of balls by using two colors or symbols. See Figure 4.3.1 for an example of a scatterplot of the sample results. **Note:** It might be valuable to graph the raw data and the median. Since there are only three trials, it would provide a nice visual connection of where the median fits into the raw data and how variable the original data are.

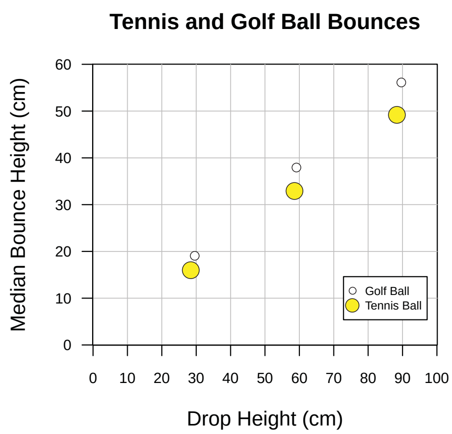


Figure 4.3.1 Scatterplot of median bounce height versus drop height for a tennis ball and golf ball

5. Ask your students to describe any patterns they observe in their scatterplots. Students should be able to use words such as positive or negative relationship. A positive relationship means that data points go from the lower left of a scatterplot to the upper right, whereas a negative relationship means the data points go from the upper left of a scatterplot to the lower right.

6. Ask your students to calculate the ratio of the bounce height to the drop height for each drop height and for both balls. Record the answers in a

table similar to Table 4.3.7 (template available on the CD), which shows the ratio of bounce height to drop height for the sample data.

Table 4.3.7 Ratios of Bounce Height to Drop Height

Drop Height	Median Tennis Bounce Height	Ratio Tennis Bounce Height to Drop Height	Median Golf Bounce Height	Ratio Golf Bounce Height to Drop Height
30	16	16/30= 0.53	19	19/30= 0.63
60	33	33/60 = 0.55	38	38/60 = 0.63
90	49	49/90 = 0.54	56	56/90 = 0.62

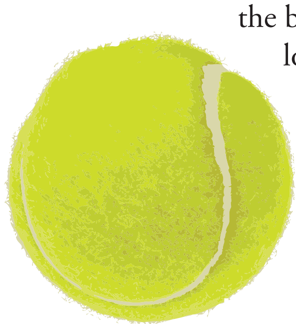
7. Ask your students to interpret the ratio of the bounce height to the drop height for the tennis ball and golf ball. Note that students' responses should center on a tennis ball bounces back around 54% of the height from which it was dropped. The golf ball bounces back more, somewhere around 63% of the height.

Interpret the Results in the Context of the Original Question

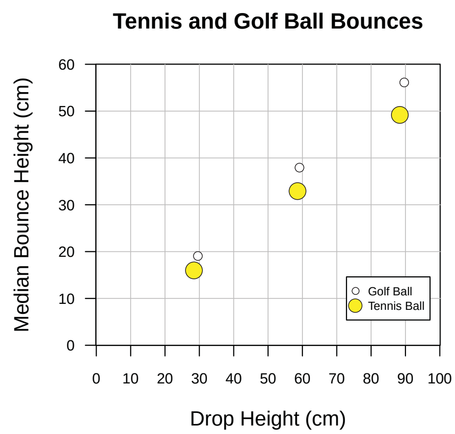
Ask your students to discuss in their groups their answer to the question, "Does the height from which a ball is dropped affect how far it bounces?" Have each group write a summary of the experiment that starts with stating an answer to the question and then supporting their answer by using their analysis. Your students should base their answer on the data collected, key calculations, their scatterplot, and the ratios.

Example of 'Interpret the Results'

Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.



We think the height from which a ball is dropped does and does not affect the bounce height too much. It depends on what you are looking at. If it's the actual height of the bounce, then it goes up if the drop height goes up. But if it's the ratio of the bounce height to the drop height, then the ratio is constant for a tennis ball or a golf ball—about .54 for the tennis ball and.63 for the golf ball. Our conclusion is based on data we got from dropping a tennis ball three times each from heights of 30, 60, and 90 cm. We dropped the ball three times from each height to get an accurate result. We then took the median of the three data points to represent the bounce height for each drop height. We did the same thing for a golf ball. Here is our scatterplot of median bounce height for each drop height:



Looking at the scatterplot, we see there is a positive relationship between drop height and bounce height for both tennis and golf balls. We also see that the relationship is pretty linear for both the tennis and the golf ball and that the golf ball bounces higher than the tennis ball at each drop height. The gap between the heights gets wider as the drop gets higher. We now see why our teacher asked us to calculate the ratio of bounce height to drop height. Here are the calculations:

Drop Height	Median Tennis Bounce Height	Ratio Tennis Bounce Height to Drop Height	Median Golf Bounce Height	Ratio Golf Bounce Height to Drop Height
30	16	16/30= 0.53	19	19/30= 0.63
60	33	33/60 = 0.55	38	38/60 = 0.63
90	49	49/90 = 0.54	56	56/90 = 0.62

It's interesting to see that the tennis ball bounces back around 54% of its drop height and the golf ball does better—at around 63%. It's probably because of the composition of a golf ball. We wonder what ratio a "super ball" would have.

Assessment with Answers

A group of students conducted the ball drop experiment using a basketball. Table 4.3.8 contains the results of their experiment when they dropped a basketball from 30, 60, and 90 cm.

Table 4.3.8 Results of Dropping a Basketball

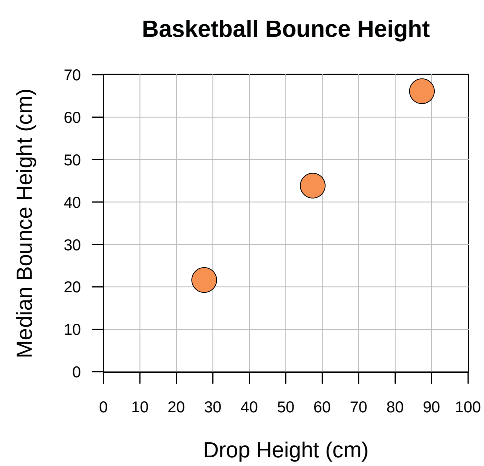
	Basketball		
	30 cm	**60 cm**	**90 cm**
Trial 1	22.0	45.0	67.0
Trial 2	23.0	44.0	68.0
Trial 3	22.0	44.0	67.0
Mean	22.3	44.3	67.3
Median	22.0	44.0	67.0
Ratio	0.73	0.73	0.74

1. Find the mean and median bounce height for each drop height and record them in the chart above.

2. Find the ratio of the median bounce height to the median drop height and record them in the chart above.

3. Discuss how the mean and median bounce heights relate to the drop height. Include the ratio of median bounce height to median drop height in your discussion. The height that the basketball bounces is about 73% of the height from which it was dropped.

4. Construct a scatterplot that shows the relationship between the heights from which the basketball was dropped and the median height of the bounce.



5. Describe the graph and the relationship between drop height and median bounce height. The higher the ball is dropped, the higher the bounce.

6. Is the bounce height of a basketball higher than either the tennis ball or golf ball that you used in the investigation? Explain your answer. Based on the sample data given in the investigation, the bounce height of the basketball was higher than both the tennis and golf ball. The median bounce height at the drop heights of 30 cm, 60 cm, and 90 cm were all higher for the basketball than both tennis and golf ball. Based on your class data collected, the answer may differ.

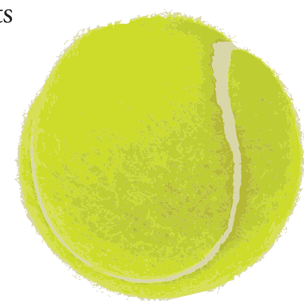
Alternative Assessment

Have your students find a ball at their house and replicate what was done in class with a family member. Have them drop the ball from 30, 60, and 90cm. They should record their data, find the mean and median, and create a scatterplot. Students should describe the relationship between drop heights and bounce height and compare their results with the results from the class experiment.

Extensions

1. Ask your students how high they think the ball would bounce if it were dropped from 25 cm? 75cm? 150 cm? Ask them what they are basing this on? After they make their predictions based on the collected data, have them drop the balls from 25, 75, and 150 cm and compare the results to their predictions. Note that predictions should not be made outside the interval of the original data. However, this should make for an interesting extension, such as seeing if the data trend changes at a certain point.

2. Ask your students how they would carry out this experiment if they wanted to find the ratio of the table tennis ball bounce height to its drop height. Their description should include what drop heights they would use, how many drops from each height, and how they would calculate their answer.



3. Have your students look at their scatterplots. Ask them if it would be possible to imagine a line being drawn to represent the trend in our data? How would drawing this line help us answer the two questions above?

References

Bereska, C., L. C. Bolster, C. A. Bolster, and R. Schaeffer. 1998. *Exploring statistics in the elementary grades: Book one, grades k–6*. Lebanon, IN: Dale Seymour.

Franklin, C., G. Kader, D. Mewborn, J. Moreno, R. Peck, M. Perry, and R. Scheaffer. 2007. *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-k–12 curriculum framework*. Alexandria, VA: American Statistical Association. www.amstat.org/education/gaise.

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Common Core State Standards for Mathematics. www.corestandards.org.

Investigation 4.4
Can You Roll Your Tongue?

Overview

This investigation focuses on students examining a relationship or association between two **categorical variables**. Specifically, they will investigate whether there is an association between gender and whether a person can roll their tongue. As part of this investigation, students will collect, organize, and analyze data in a **two-way table**; construct and analyze **segmented bar graphs**; and calculate the **percentages** of boys and girls who can roll their tongue. This investigation is based on an activity in *Probability Through Data*, a module in the Data-Driven Mathematics series (1999).

GAISE Components

This investigation follows the four components of statistical problem solving put forth in the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report.* The four components are formulate a statistical question that can be answered with data, design and implement a plan to collect appropriate data, analyze the collected data by graphical and numerical methods, and interpret the results of the analysis in the context of the original question. This is a GAISE Level B activity.

Learning Goals

Students will be able to do the following after completing this investigation:

- Organize data collected into a two-way table
- Analyze data in a two-way table

Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Common Core State Standards Grade Level Content

6RP3c Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

6SP3 Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers.

8.SP.4 Understand that patterns of association also can be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Principles and Standards for School Mathematics

Data Analysis and Probability

Grades 6-8 Students should understand and use ratios and percentages to represent quantitative relationships and formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population.

Materials

- Data collection sheet (available on the CD)
- Data recording sheet (available on the CD)
- Grid paper
- Color markers

Estimated Time

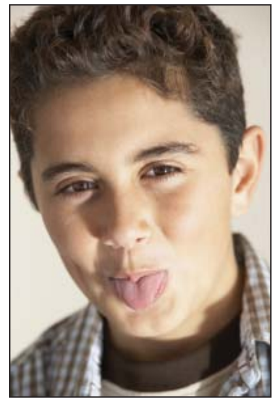
One day

Instructional Plan

✏ Formulate a Statistical Question

Ask your students to look around their classroom. Pose the question, “Is anyone in the room exactly like you?” “Are identical twins exactly the same?” Discuss that there are many traits or characteristics that make us different from each other. Have students list some of these traits. Examples are hair color, eye color, skin color, blood type, having double-jointed elbows, having "free" earlobes or "attached" earlobes, and whether they can roll their tongue. Discuss with your students that many of these traits are genetic (i.e., inherited or passed on from their parents). Ask which of the traits might have been inherited from their parents.

Tell students there are many traits they could investigate. Indicate that, for this activity, they will be investigating rolling one's tongue (even though it isn't genetic). The statistical question is, "How do boys and girls compare regarding the ability to roll their tongues?"


Can't roll tongue


Can roll tongue

Collect Appropriate Data

1. Have one student demonstrate how he/she is able to roll his/her tongue and another demonstrate that he/she is unable to roll his/her tongue.

2. Hand out a data collection sheet to each student. Your students should check whether they are a boy or girl and whether they can roll their tongue. Collect each of the data collection sheets. Figure 4.4.1 is an example of a data collection sheet.

Boy ________

Girl ________

Can roll tongue ________

Can't roll tongue ________

Figure 4.4.1 Data Collection Sheet

3. Hand out a recording sheet (available on the CD) to each student. Take each of the data collection sheets and read whether the sheet is checked boy or girl and whether the student can roll their tongue. As you read each data collection sheet, students should record the data on the recording sheet as shown in Table 4.4.1. Suggest that they write B for boy, G for girl, Y for yes they can roll their tongue, and N for no they cannot roll their tongue.

Table 4.4.1 Example of Class Recording Sheet

Student	Boy or Girl	Roll Your Tongue Yes or No?
1	B	N
2	B	Y
3	G	Y
…	…	…

Analyze the Data

1. Discuss with your students how one way to help analyze the data is to organize it into a table. Ask them what answers they could record when they were reading the data collection sheets. On the board, display Table 4.4.2. Ask your students to fill in the frequencies (counts) for the four possibilities based on their recording sheet.

Table 4.4.2 Frequency Table

Possibilities	Frequency
Boy – Yes	
Boy – No	
Girl – Yes	
Girl – No	
Total	

2. Explain to your students that their frequency table can be displayed in a different way, called a two-way table. A two-way table organizes data about two categorical variables with rows labeled with the categories of one variable and the columns labeled with the categories of the other variable. In this investigation, the rows of the table are labeled with gender—boys and girls—and the columns are labeled with whether a person can roll their tongue. Demonstrate drawing and labeling the two-way table. The general form is shown in Table 4.4.3. Note that the two-way format is useful when investigating whether there is a relationship between two categorical variables.

Table 4.4.3 Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy			
Girl			
Total			

3. Label each cell in Table 4.4.3 with letters representing frequencies, as shown in Table 4.4.4.

Table 4.4.4 Example of Completed Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	a	b	
Girl	c	d	
Total			

4. Explain to your students that the cell labeled "a" will contain the number of students who are both a boy and who said they could roll their tongue. Ask your students what the cell labeled "b" represents. Cell "c"? Cell "d"?

5. Ask your students how many boys are in the sample symbolically, according to Table 4.4.4? **Note:** There are "a+b" boys. How many girls? There are "c+d." How many students can roll their tongues? "a+c" can roll their tongues. How many can't? "b+d" can't.

6. Have your students fill in the two-way table based on their class day as recorded in their frequency table, Table 4.4.2. An example of what their table may look like is given in Table 4.4.5.

Table 4.4.5 Row of the Boys' Data from the Two- Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	**15**
Girl	6	4	**10**
Total	**14**	**11**	**25**

7. Ask your students to use Table 4.4.5 to answer the following questions. As students answer each question, have them point to the appropriate cell.

   a. How many students were in the class?

   b. How many students could roll their tongue?

   c. How many students were girls?

   d. How many students were boys?

   e. How many girls could roll their tongue?

   f. How many boys could roll their tongue?

   g. How many boys could not roll their tongue?

8. Ask your students to put their percentages in a two-way table. See Table 4.4.8.

9. Ask your students to find the percentage of boys who could roll their tongue. To help them answer this question, show them only the row with the boys' data. See Table 4.4.6. Have them find the fraction that answers the question, convert it to a decimal, and then convert it to a percentage. For example, for the boys who can roll their tongue, 8/15 = .53 = 53%.

Table 4.4.6 Row of the Boys' Data from the Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	15

10. Using Table 4.4.7, ask your students to find the percentage of girls who can and cannot roll their tongue.

Table 4.4.7 Row of the Girls' Data from the Two-Way Table

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Girl	6	4	10

11. Ask your students to put their percentages in a two-way table. See Table 4.4.8. Note that the Total row percentages are each 100%.

Table 4.4.8 Example of Row Percentages

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8/15 = .53 = 53%	7/15 = .47 = 47%	15/15 = 1.00 = 100%
Girl	6/10 = .60 = 60%	4/10 = .40 = 40%	10/10 = 1.00 = 100%
Total			

12. To help your students visualize the different percentages of boys and girls who can and cannot roll their tongue, demonstrate the construction of a segmented bar graph. Using Table 4.4.8, a segmented bar graph is shown in Figure 4.4.2. Note that the percentages could also be visualized in side-by-side bar graphs.

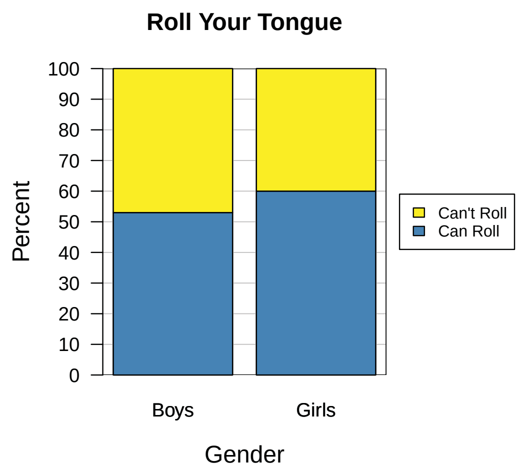


Figure 4.4.2 Segmented bar graph of example class data

13. Recall the question and ask your students to provide an answer to "How do boys and girls compare regarding the ability to roll their tongues?" Note that if they fell into the "count" trap previously, they can now see that the answer is a higher percentage of girls can roll their tongues than boys, with a percentage of 60% as compared to 53% for boys.

Interpret the Results in the Context of the Original Question

1. Have your students recall the original statistical question, "How do boys and girls compare regarding the ability to roll their tongues?" Have each group of students write an answer to the question and then justify it using the two-way table, appropriate calculations involving percentages, and the segmented bar graph. Suggest to the students that they should focus on the difference in the percentages and the heights of the bars in the segmented bar graph. If the percentages are relatively close, we would conclude there is no difference between the percentage of boys and girls who can roll their tongue. Have each group of students present their results to the class.

Example of 'Interpret the Results'

Note: The following is not an example of actual student work, but an example of all the parts that should be included in student work.

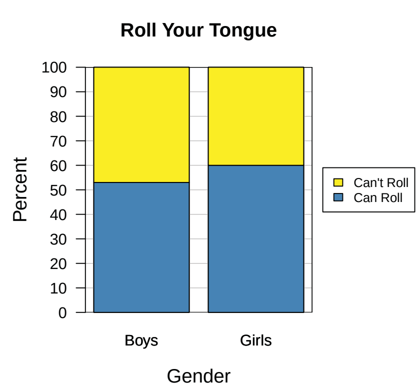
In our biology class, we often talk about genetics, so we thought a good statistics project in our mathematics class would be to take a genetic trait and see if there is any difference in boys and girls. We chose rolling our tongues. (After our study was complete, we found out that rolling one's tongue is not actually genetic. It is a learned trait. But it was fun doing the experiment anyway.) Our statistical question was "How do boys and girls compare regarding the ability to roll their tongues?" We collected data by making a list of boys or girls and whether they could roll their tongue. We then counted how many there were in each of the four categories and organized the data in a two-way table like this one.

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8	7	15
Girl	6	4	10
Total	**14**	**11**	**25**

So, to answer the question, some of us say boys are more likely to roll their tongues than girls are. But, we messed up because there were more boys in class than girls. So, we should be looking at percentages, not counts. When we calculated the percentages, we almost based them on 25, but realized they had to be calculated within boys' and girls' totals. So, here is our table of row percentages.

	Yes – Can Roll Tongue	No – Can't Roll Tongue	Total
Boy	8/15 = .53 = 53%	7/15 = .47 = 47%	15/15 = 1.00 = 100%
Girl	6/10 = .60 = 60%	4/10 = .40 = 40%	10/10 = 1.00 = 100%
Total			

The actual answer to our question is that a higher percentage of girls can roll their tongues as compared to boys. Sixty percent of girls could roll their tongues compared to 53% of boys. Our teacher showed us how to visualize these results in what is called a segmented bar graph. It makes it clear that the percentage of girls is higher.



Assessment with Answers

A survey asked a group of students if they participated in a sport and if they played a musical instrument. Table 4.4.7 shows the survey results.

Table 4.4.7 Survey Results

	Music Yes	Music No	Total
Sport Yes	18	2	20
Sport No	8	22	30
Total	**26**	**24**	**50**

Use the table to answer the following questions:

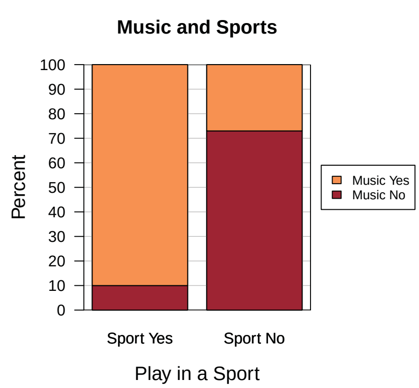
1. How many students said they participated in a sport? Twenty said they participated in a sport.

2. How many students said they did not play a musical instrument? Twenty-four said they did not play a musical instrument.

3. What does the number 8 represent in the table? The number 8 represents the number of students who said no to sports and yes to music.

4. What percentage of those who said they participated in a sport also played a musical instrument? 18/20 = .90.
5. What percentage of those who said they did not participate in a sport played a musical instrument? 8/30 = .27.

6. If a student participates in a sport, are they more likely to play a musical instrument than a student who does not participate in a sport? Use words, numbers, and graphs to explain your answer.



Students who do not participate in a sport are much less likely to play a musical instrument than those who do participate in a sport. Twenty-seven percent of students who do not participate in a sport also played an instrument while 90% of those that did participate in a sport played an instrument. The segmented bar graph shows the big difference between the groups who do and don't play sports and whether they play an instrument.

Right-thumbed

Left-thumbed

Extensions

1. Ask students to collect data at home. Each student should ask one parent/guardian if he/she could roll his/her tongue. Collect data in a table during the next class period:

Possibilities	Number
Student yes – Parent/guardian yes	
Student yes – Parent/guardian no	
Student no – Parent/guardian yes	
Student no – Parent/guardian no	

   Your students should organize the data in a two-way table. Based on the table and calculated percents, students should determine if there appears to be a relationship between whether the parent/guardian can roll his/her tongue and whether the student can roll his/her tongue.

2. Your students could investigate if there appears to be a relationship between whether a person is left-handed or right-handed and whether they are left-thumbed or right-thumbed. **Note:** To determine whether one is left- or right-thumbed, have your students clasp their hands together immediately without thinking about it. Then look at the pictures to the left to determine the category. Students could collect class data and analyze the data to determine if there appears to be a relationship.

References

Franklin, C., G. Kader, D. Mewborn, J. Moreno, R. Peck, M. Perry, and R. Scheaffer. 2007. *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-k–12 curriculum framework*. Alexandria, VA: American Statistical Association. www.amstat.org/education/gaise.

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