2021 BCCTM Algebra Questions (4-08-21)

1. A father and son are buying hot dogs and lemonade for a family picnic. They have only a $20 bill to spend. Lemonade costs $3.50 per bottle and they must buy one bottle. Hot dogs cost $2.50 per package. What is the maximum number of packages of hot dogs they can buy?

1. **ANSWER: 6**

   Let x represent the number of packages of hot dogs. Then the problem conditions are that

$$\begin{aligned} 2.50x + 3.50 &\leq 20 \\ 2.50x &\leq 16.50 \\ x &\leq 16.50/2.50 = 6.6 \end{aligned}$$

   Since x must be an integer, $x = 6$.

2. It is Tim's 4th birthday and he has a cake with 4 candles on it. Tim makes a wish and tries to blow out the candles. Every time he blows, Tim blows out some number of candles between 1 and the number that remain, including both 1 and that number. How many ways are there for Tim to blow out the candles on the cake?

2. **ANSWER: 8**

   Enumerate the possible sequences of candles blown out at each try in a systematic way:

   (a) 4;
   (b) 3, 1
   (c) 2, 2
   (d) 2, 1, 1
   (e) 1, 3
   (f) 1, 2, 1
   (g) 1, 1, 2
   (h) 1, 1, 1, 1

   There are 8 possible ways to blow out the candles.

3. What is the result of "one-eighth divided by one-sixteenth"?

   (A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{2}$ **(D)** 2 **(E)** 4

3. **Answer (D):**

   The statement "one-eighth divided by one-sixteenth" is

$$\frac{\frac{1}{8}}{\frac{1}{16}} = \frac{16}{1} \cdot \frac{1}{8} = 2.$$

4. A whole ham costs \$2.00 per pound and a spirally-cut-ham costs \$3.00 per pound. If a whole ham weights 18.3 pounds and costs \$7 more than a spirally-cut-ham, to the nearest tenth, what does the spirally-cut-ham weigh?

4. **ANSWER: 9.9**

   Let x be the weight of the spirally-cut-ham. Then $3x = 2(18.3) - 7 = 29.6$. Then $x = \frac{29.6}{3} = 9.86$. Answer: 9.9. Note to self: buy the whole ham.

5. Sheila bought 100 shares of Sweet Sugar at \$49.50 per share and paid a 2% commission as well. Financial irregularities were brought to light and the stock started a precipitous decline. Sheila sold all 100 shares at \$29 per share and again paid a 2% commission. Including what she paid for commission, what was her total loss?

5. **ANSWER: 2207**

   She paid $100 \cdot 49.50 + 100 \cdot 49.50 \cdot 0.02 = 5049$ dollars. She paid $0.02 \cdot 2900 = 58$ dollars commission when she sold so she received $2900 - 58 = 2842$ dollars. She lost $5049 - 2842 = 2207$ dollars.

6. In weaving an oriental carpet, a weaver can tie one knot every 5 seconds. If a rug has 10 knots per horizontal inch and 9 knots per vertical inch, how long in hours would it take to weave one square foot?

6. **ANSWER: 18**

   1 knot per 5 seconds is 12 knots per minute. Multiplying by 60 gives 720 knots per hour. There are 90 knots per square inch and 144 square inches in a square foot. The number of knots per square foot is 90 times 144 so the time in hours is

$$\frac{90 \cdot 144 \text{ knots/square foot}}{720 \text{ knots/hour}} = 18 \frac{\text{hours}}{\text{square foot}}.$$

   Answer: 18.

7. There is a positive Fahrenheit temperature F at which the corresponding Celsius temperature C is negative but $|C| = F$. If F can be written in simplest terms as $\frac{a}{b}$ where a and b have no common divisors, what is $a + b$?

7. **ANSWER: 87**

   Since $C = \frac{5}{9}(F - 32)$ then $-F = \frac{5}{9}(F - 32)$ or $\frac{14}{9}F = \frac{5}{9} \cdot 32$ so $F = \frac{80}{7}$. The sum is 87.

4. What is the simplified value of

$$\sqrt{\frac{0.\overline{81}}{0.\overline{09}}}$$

where $0.\overline{ab}$ represents the repeating decimal $0.ababab\ldots$.

(A) $\frac{1}{3}$ **(B)** $\frac{\sqrt{3}}{9}$ **(C)** 3 **(D)** $3\sqrt{3}$ **(E)** $\frac{3}{10}$

4. **Answer (C):**

Let $N = 0.\overline{81}$, then $100N = 81.\overline{81}$. Subtracting the first from the second gives $99N = 81$, making $N = \frac{9}{11}$. In similar fashion, if $M = 0.\overline{09}$, then $M = \frac{1}{11}$. Thus, we have

$$\frac{\sqrt{\frac{9}{11}}}{\sqrt{\frac{1}{11}}} = \frac{3}{\sqrt{11}} \cdot \frac{\sqrt{11}}{1} = 3.$$

Answer: (C)

6. Which of the following numbers in the answer choices is prime?

(A) $30^2 - 60 + 1$ **(B)** $30^2 + 60 + 1$

(C) $30^2 + 90 + 1$ **(D)** $30^2 - 30 + 1$

(E) $30^2 + 30 + 1$

6. **Answer (C):**

(A) is $(30 - 1)^2$ and (B) is $(30 + 1)^2$ so neither is prime. (D) is 871 which equals $13 \cdot 67$ and (E) is 931 which equals $7 \cdot 133$ so neither is prime. That leaves (C) which equals 991 which is prime. Answer: (**C**).

7. The ratio of the measures of the angles of a triangle is $9 : 5 : 6$. Let 15° be added to the smallest angle and 6° be added to the second smallest angle and create a new triangle using these two angles. What is the ratio of the angles of the new triangle?

(A) $9 : 5 : 6$ **(B)** $10 : 6 : 7$ **(C)** $1 : 2 : 3$

(D) $1 : 1 : 1$ **(E)** $3 : 2 : 1$

7. **Answer (D):**

Since the ratio is $9 : 5 : 6$, there are 20 parts, with one angle having $9/20$ of the total, a second angle having $5/20$ of the total and the third angle having $6/20$ of the total. That is, from the total of 180°, the angles are 81°, 45° and 54° respectively. Adding 15° and 6° to the smallest and second smallest angles makes 60° and 60°. The third angle of the new triangle is 60°, making an equilateral triangle. The ratio of the angles is $1 : 1 : 1$ and the answer is (D).

1. At a meeting of 12 business people, the activities begin with each person shaking every other person's hand exactly once. How many handshakes take place?

1. **ANSWER: 66**

   Each of the 12 people shakes the hand of 11 people. But that counts each handshake twice, so there are $12 \cdot 11/2 = 66$ handshakes total.

2. Bob eats like a bird. He eats one grain of rice at a time and it takes him 0.20 seconds to eat one grain. He needs to eat 550 grams per day and there are 48 grains of rice in a gram. In any given day he spends A hours and B minutes ($0 \leq B < 60$) eating to reach 550 grams. What is $A+B$?

2. **ANSWER: 29**

$$550\frac{\text{grams}}{\text{day}} \cdot 48\frac{\text{grains}}{\text{gram}} = 26{,}400\frac{\text{grains}}{\text{day}}.$$

$$26{,}400\frac{\text{grains}}{\text{day}} \cdot 0.2\frac{\text{seconds}}{\text{grain}} = 5280\frac{\text{seconds}}{\text{day}}.$$

$$\frac{5280 \text{ seconds}}{60 \text{ seconds/min}} = 88 \text{ minutes}.$$

$$88 \text{ minutes} = 1 \text{ hour and } 28 \text{ minutes}$$

Thus $A = 1$ and $B = 28$ so $A + B = 29$.

3. Tim received the following scores on his math tests, listed in increasing order 86, 88, 90, 91, 92, 93, 93, 95, 100. Three of these scores can be removed without affecting the average. What is the sum of those three scores?

3. **ANSWER: 276**

   The mean is 92, so the mean and balanced pairs about the mean can be removed without changing the mean. That would be 91, 92, 93. The sum is $91 + 92 + 93 = 276$.

1. In scientific notation, how many minutes are in 35 weeks?

   (A) 3.528×10^3 **(B)** 3.528×10^4 **(C)** 3.528×10^5 **(D)** 3.528×10^6 **(E)** 3.528×10^7

1. **Answer (C):**

$$35 \text{ weeks} \cdot 7\frac{\text{days}}{\text{week}} \cdot 24\frac{\text{hours}}{\text{day}} \cdot 60\frac{\text{minutes}}{\text{hour}} = 352{,}800 \text{ minutes}$$

$$= 3.528 \times 10^5 \text{ minutes}.$$

5. A piece of string 60 inches long is cut into 5 pieces, with each piece 2 inches longer than the previous piece. What is the length of the longest piece?

5. **ANSWER: 16**

Let the length in inches of the shortest piece be x, so the 5 pieces have lengths x, $x+2$, $x+4$, $x+6$ and $x+8$. Then

$$x + (x+2) + (x+4) + (x+6) + (x+8) = 5x + 20 = 60$$

and $x = 8$, so $x + 8 = 16$.