2021 BCCTM Geometry Questions (4-08-21)

1. If the surface area of a cube equals its volume, what is the sum of the lengths of all the edges of the cube?

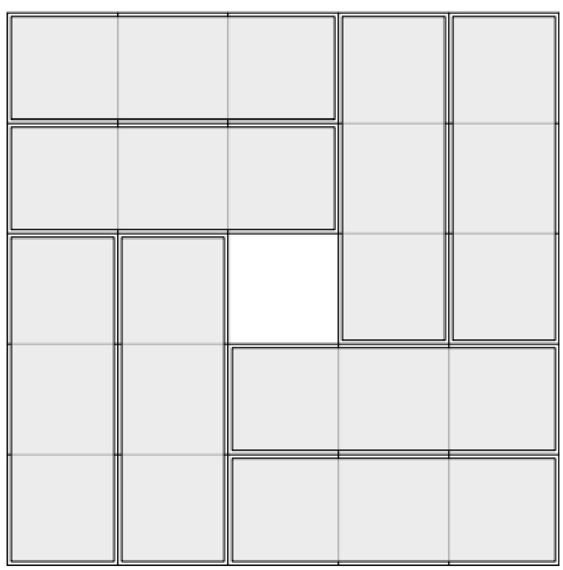
1. **ANSWER: 72**

   Let x be the length of the edge of the cube. Then $x^3 = 6x^2$ so $x = 6$. There are 12 edges so the sum of the lengths equals $12 \cdot 6 = 72$.

2. A 5 by 5 square is divided up into 25 unit squares. It will be covered with 1 by 3 tiles and 1 by 1 tiles with no overlaps. What is the least number of 1 by 1 tiles that can be used?

2. **ANSWER: 1**

   Since $8 \cdot 3 = 24 = 25 - 1$ and $9 \cdot 3 = 27 > 25$ it seems likely that we can tile the square using 8 1 by 3 tiles and no more than just one 1 by 1 tile. The figure shows that the least number of 1 by 1 tiles is indeed 1.

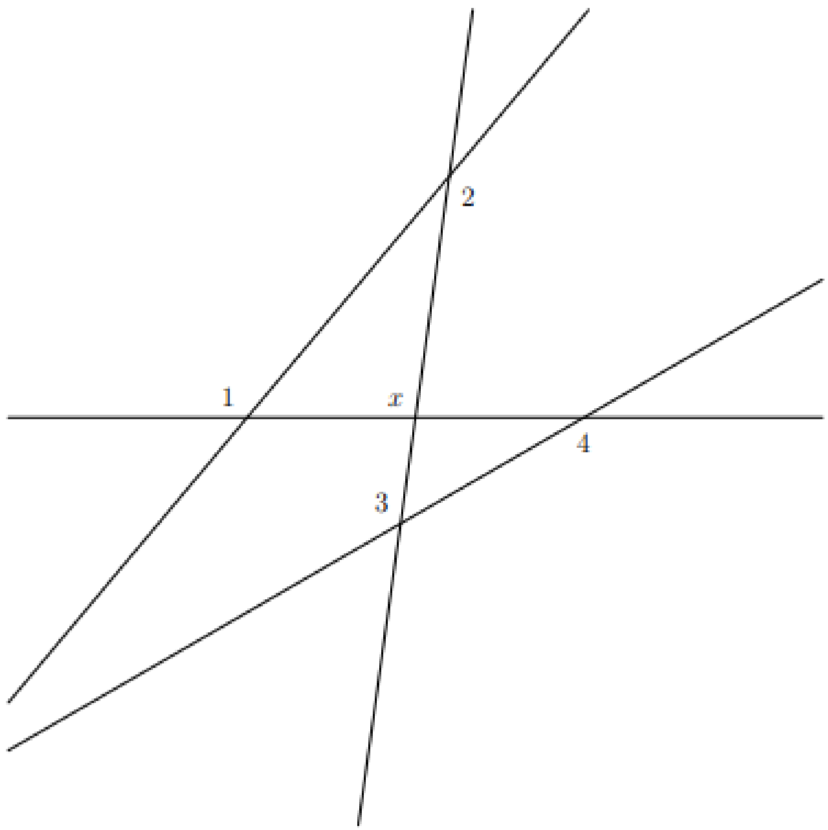


3. Given the system $3x + 13y = 11$ and $7x + 5y = 19$, what is the value of $5x - 10y$?

3. **ANSWER: 10**

   Subtract the first from the second obtaining $4x - 8y = 8$ which is $x - 2y = 2$. Then multiply by 5 getting $5x - 10y = 10$.

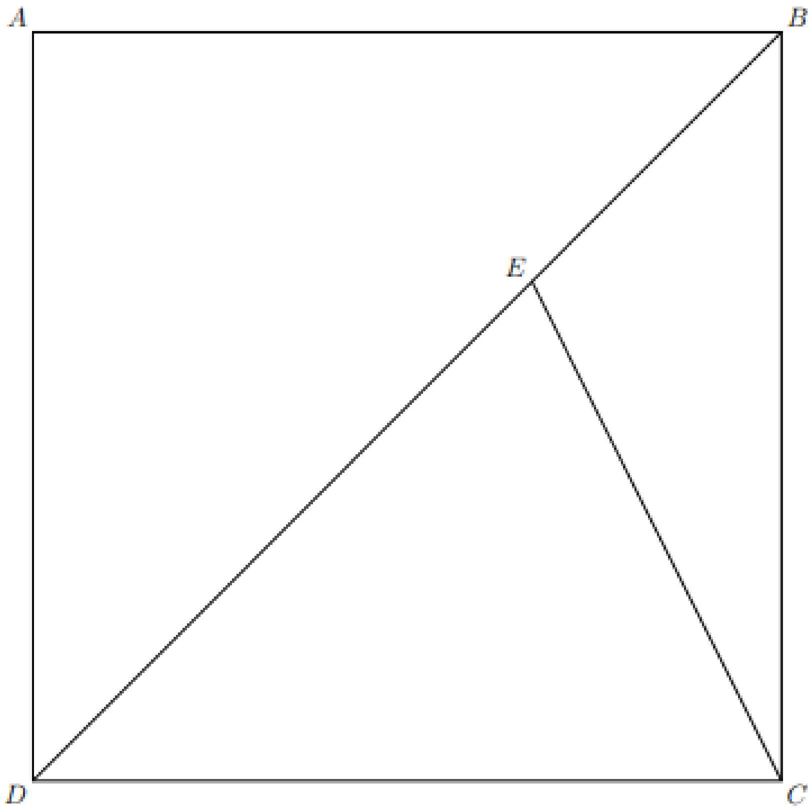
4. What is the value of x in degrees such that $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 556°$?



4. **ANSWER: 98**

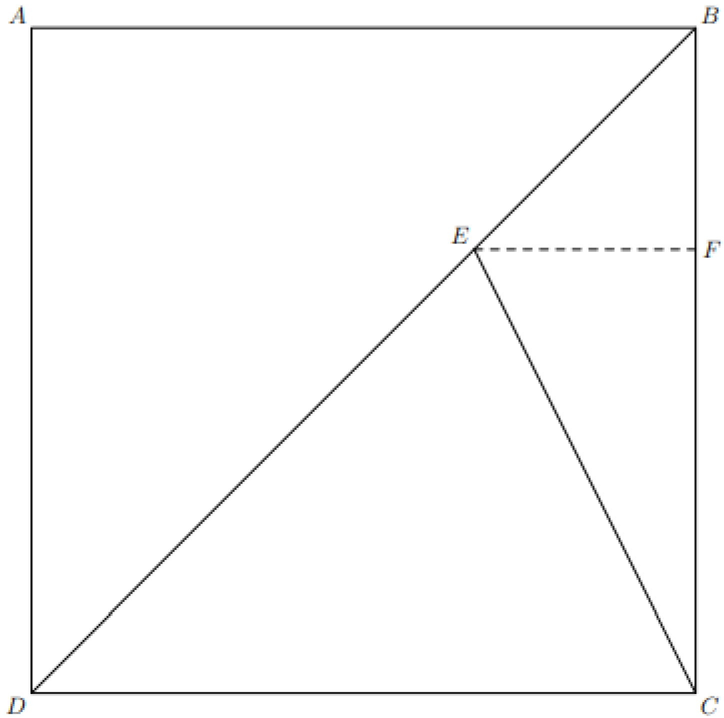
   Since an exterior angle of a triangle is the sum of the two remote interior angles, $m\angle 1 = x + (180° - m\angle 2)$ so $m\angle 1 + m\angle 2 = x + 180°$ and similarly, $m\angle 3 = x + (180° - m\angle 4)$. Then $m\angle 3 + m\angle 4 = x + 180°$. Thus $2x + 360° = 556°$, $2x = 196°$ and $x = 98°$.

6. Square $ABCD$ has sides of length 6 and the length of $\overline{BE}$ equals $2\sqrt{2}$. The length of $\overline{EC}$ can be written as $a\sqrt{b}$ where a and b are integers and b has no perfect square factors. What is $a+b$?



6. **ANSWER: 7**

Draw $\overline{EF}$ perpendicular to $\overline{BC}$. Then $\triangle BFE$ is a 45°–45°–90° right triangle and since $BE = 2\sqrt{2}$, $BF = EF = 2$ and so $FC = 4$. Then $EC^2 = 2^2 + 4^2 = 20$, so $EC = 2\sqrt{5}$. Then $a = 2$, $b = 5$ and $a + b = 7$.



5. A piece of string 60 inches long is cut into 5 pieces, with each piece 2 inches longer than the previous piece. What is the length of the longest piece?

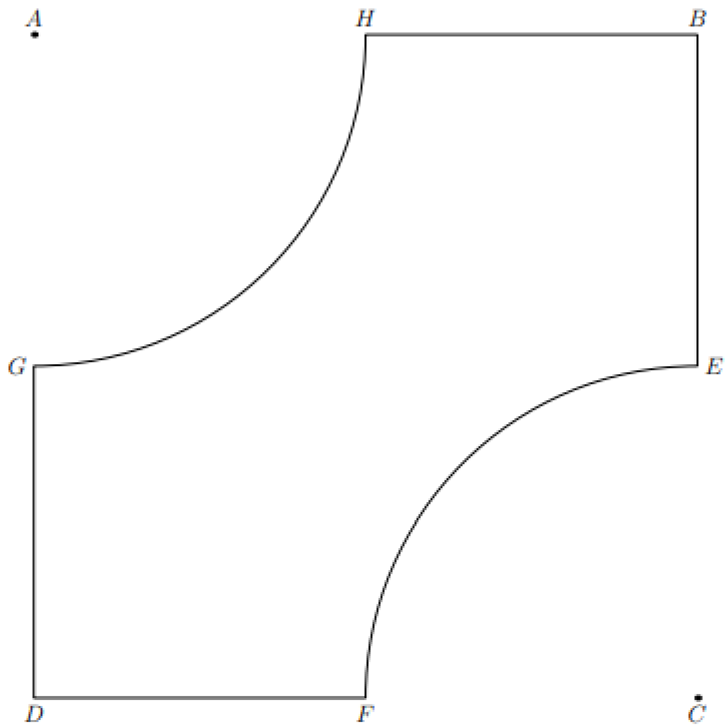
5. **ANSWER: 16**

Let the length in inches of the shortest piece be x, so the 5 pieces have lengths x, $x+2$, $x+4$, $x+6$ and $x+8$. Then

$$x + (x+2) + (x+4) + (x+6) + (x+8) = 5x + 20 = 60$$

and $x = 8$, so $x + 8 = 16$.

7. Square $ABCD$ has side length 2. Quarter-circles of radius 1 are cut from the square at A and C, leaving the figure $BEFDGH$ pictured below. What is the area of figure $BEFDGH$?

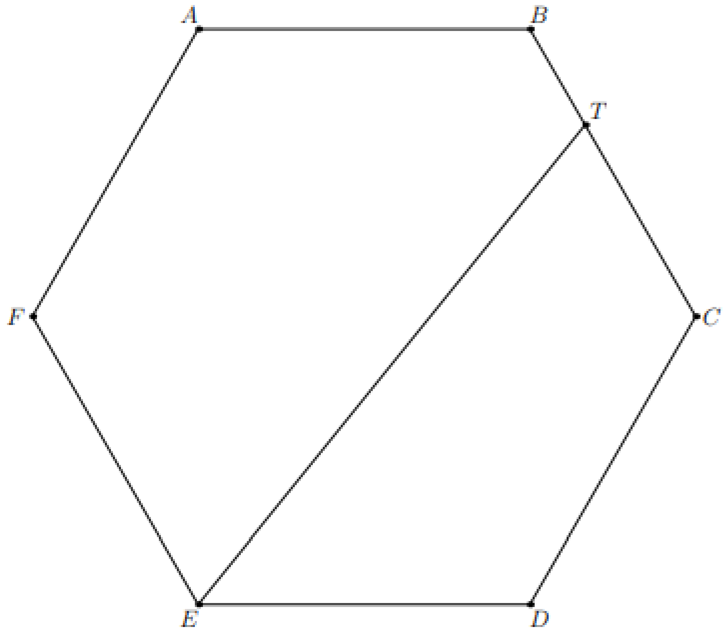


(A) $4-\pi$ **(B)** $\pi/2$ **(C)** $4-\pi/2$ **(D)** 3 **(E)** $2+\pi/2$

7. **Answer (C):**
The square has area $2 \cdot 2 = 4$. Each quarter-circle has area $\frac{1}{4} \cdot \pi \cdot 1^2 = \frac{\pi}{4}$,

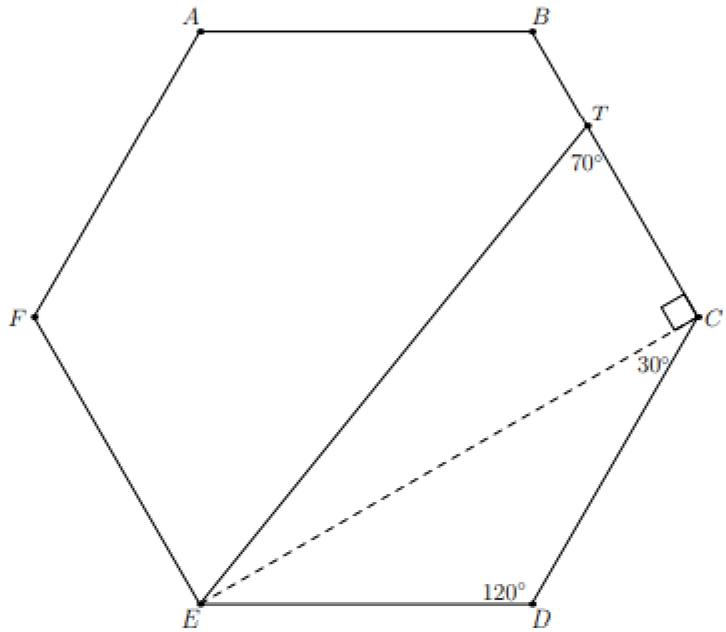
so the total area of the two quarter-circles removed is $\frac{\pi}{2}$. The remaining area is $4 - \frac{\pi}{2}$.

8. $ABCDEF$ is a regular hexagon. If $\angle ETC = 70^\circ$, what is the measure of $\angle TED$?



8. **ANSWER: 50**

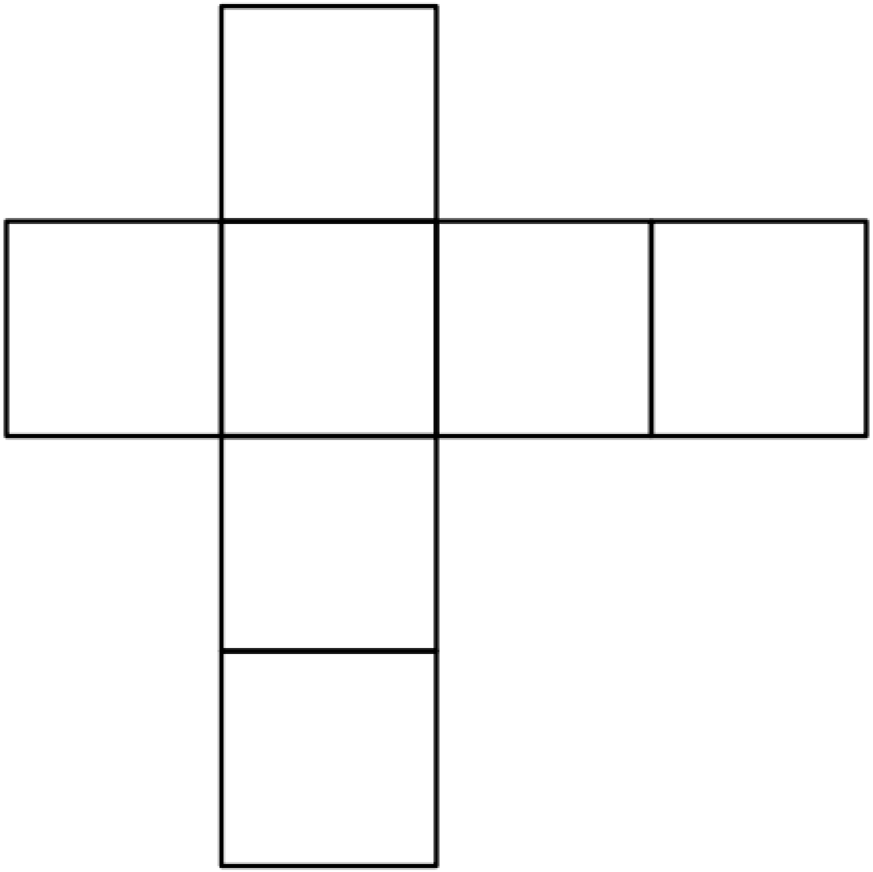
Draw $\overline{EC}$. Since $\angle EDC = 120^\circ$ and $\triangle EDC$ is isosceles, $\angle ECD = \angle CED = 30^\circ$, making $\angle ECT = 90^\circ$ so $\angle TEC = 20^\circ$, making $\angle TED = \angle TEC + \angle CED = 20^\circ + 30^\circ = 50^\circ$.



OR

Since the angles of quadrilateral $TCDE$ sum to $360°$ and two of the angles are $120°$, the other two angles sum to $120°$, so $\angle TED = 120° - 70° = 50°$.

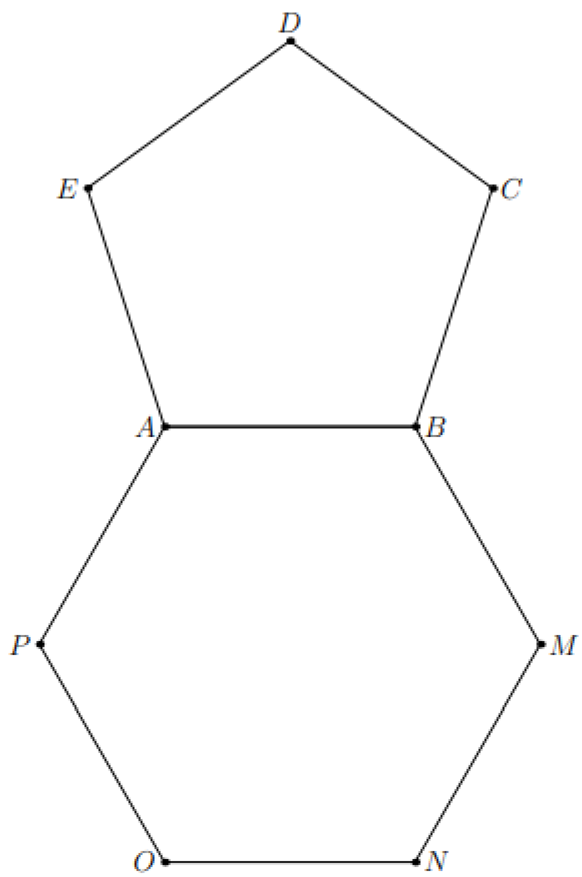
1. The accompanying figure consists of congruent squares. If the area of the figure is 63, what is its perimeter?



1. **ANSWER: 48**

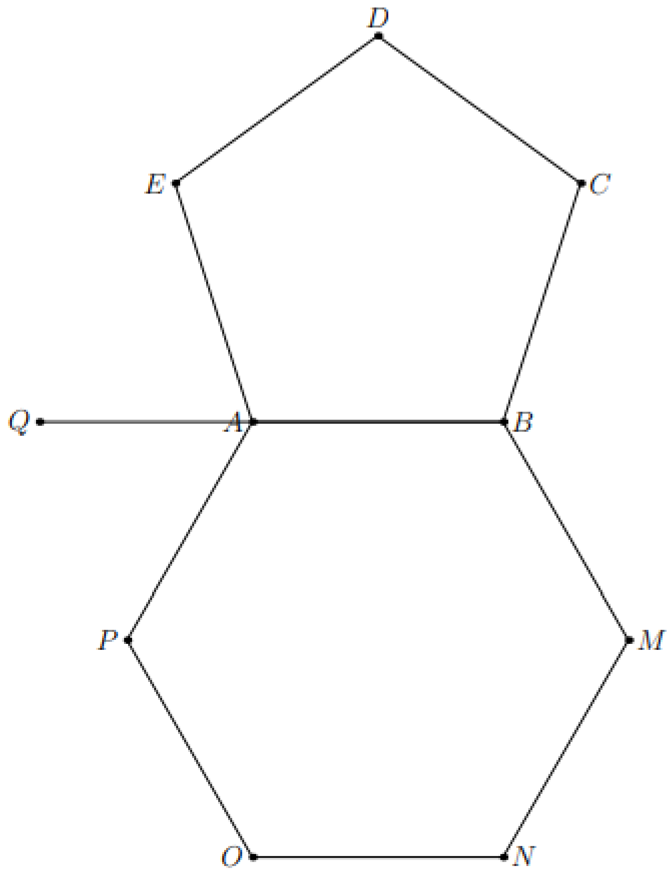
There are 7 squares of side x so $7x^2 = 63$, making $x = 3$. There are 16 sides of length 3 to the perimeter so the perimeter is $3 \cdot 16 = 48$.

3. A regular pentagon $ABCDE$ and a regular hexagon $ABMNOP$ share a common side $\overline{AB}$. What is the angle measure in degrees of $\angle EAP$?



3. **ANSWER: 132**

Consider point Q on the line $\overleftrightarrow{AB}$ with A between Q and B. Then the exterior angle EAQ has $360/5 = 72$ degrees and the exterior angle QAP has $360/6 = 60$ degrees, so the angle EAP is 132 degrees.



5. A rectangular cake 4 inches high and 12 inches by 24 inches is cut into rectangular pieces that are 3 inches wide by 6 inches long. By how many square inches has the surface area of the cake been increased?

5. **ANSWER: 864**

The surface area of the original cake is $2 \cdot 24 \cdot 4 + 2 \cdot 12 \cdot 4 + 2 \cdot 12 \cdot 24 = 864$. The newly cut pieces will have the same top, bottom, and side surface area but they pick up some additional surface. Using the left hand diagram, the corner pieces labeled A will add a 6 by 4 side and a 3 by 4 side, thereby adding $4(6 \cdot .4 + 3 \cdot 4) = 144$. Side pieces marked B will add two 4 by 6 sides and one 4 by 3 side thereby adding $4(2 \cdot 6 \cdot 4 + 1 \cdot 3 \cdot 4) = 240$. Side pieces marked C will add one 4 by 6 side and two 3 by 6 sides thereby adding $4(1 \cdot 4 \cdot 6 + 2 \cdot 4 \cdot 3) = 192$. Interior pieces marked D will add $4(2 \cdot 6 \cdot 4 + 2 \cdot 3 \cdot 4) = 288$. The total added is 864.

Using the right-hand diagram, the pieces marked A will contribute $4(4 \cdot 3 + 4 \cdot 6) = 144$ while the pieces marked B will contribute $12(2 \cdot 4 \cdot 6 + 1 \cdot 4 \cdot 3) = 720$. The total again is 864.

	6	6	6	6
3	A	C	C	A
3	B	D	D	B
3	B	D	D	B
3	A	C	C	A

	3	3	3	3	3	3	3	3
6	A	B	B	B	B	B	B	A
6	A	B	B	B	B	B	B	A

6. The four triangular faces of a right pyramid on a square base $ABCD$ are equilateral triangles of side length 1 meeting at the vertex P. The feet of the altitudes to $\triangle PAB$ and $\triangle PBC$ are Q on $\overline{AB}$ and R on $\overline{BC}$. What is the area of $\triangle PQR$?

   (A) $\sqrt{5}/8$ **(B)** $\sqrt{3}/4$ **(C)** $1/2$ **(D)** $3/4$ **(E)** $\sqrt{3}/2$

6. **Answer (A):**

   Each altitude $\overline{PQ}$ and $\overline{PR}$ has length $\frac{\sqrt{3}}{2}$, so $\triangle PQR$ is isosceles. The base is of length $QR = \sqrt{2}/2$. Then by the Pythagorean Theorem, the altitude of $\triangle PQR$ has length $\sqrt{(\sqrt{3}/2)^2 - (\sqrt{2}/4)^2} = \sqrt{3/4 - 1/8} = \sqrt{5/8} = \sqrt{10}/4$. The area of $\triangle PQR$ is then $(1/2)\cdot(\sqrt{2}/2)\cdot(\sqrt{10}/4) = \sqrt{5}/8$.

1. If the length of the radius of a circle is decreased by 20%, then the area of the circle is decreased by k%. What is the value of k?

1. **ANSWER: 36**

   The original area is πr^2. If the radius is decreased by 20%, the new radius is $\frac{4}{5}r$ and the new area would be

$$\pi\left(\frac{4}{5}r\right)^2 = \pi\frac{16}{25}r^2 = \pi\frac{64}{100}r^2.$$

   The area has been reduced by 36% so $k = 36$.